

Endogenous Social Networks in the Labor Market

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Abstract

I develop an equilibrium model of endogenous network formation. In my model workers differ in their job loss rates and networks enable workers to find jobs more frequently. It is costly for workers to help other members of their network and workers trade off costs against the benefits they will derive from more frequent job offers when they are unemployed. My model is an extension of search and matching model by Pissarides that captures social networks in the matching process and allows for heterogeneity in job loss rates. I analyze survey data from PSID with information on job finding through social networks. Consistent with the data, our model predicts that workers with higher job loss rates are more likely to use their social contacts to find jobs; they have shorter unemployment durations and lower wages. The endogenous framework allows me to analyze how networks are affected by labor market conditions. I show that a more generous unemployment benefit scheme will reduce the use of networks and there is a negative correlation between unemployment rate and the fraction of workers using social contacts to get a job. I also show that modelling social networks give us more insights on the regional inequalities in the labor market.

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1 Introduction

Social networks play a prominent role in the U.S. labor market established by empirical studies which show that between 30 and 60 percent of jobs found through friends and relatives. (Ioannides and Loury (2004), Montgomery(1991)) which starts an extensive literature which is devoted to the exploration of the importance of social networks in the labor market. An important part of this literature is dedicated to the analysis of the effects of social networks on labor market outcomes taking the the network as given.¹

The literature has mainly focused on how membership in a social network can be advantageous for labor market outcomes but little attention has been paid to the individual characteristics of workers which lead to the participation in the social networks. An incentive for participation in the network is insurance, even if an individual is employed now, there is a positive probability of becoming unemployed in future periods and having to rely on the network to gain a job. I analyze data from PSID and observe that workers with higher job loss rates are more likely to use their social networks to find a job.

In view of this empirical observation, I develop a simple equilibrium model of social networks which endogenizes the network formation in the labor market. In my model, workers differ in their job loss rates and social networks enable workers to find jobs more frequently. It is costly for workers to help other members of their network to find jobs and workers trade off the cost of helping friends against the benefits they will derive from more frequent job offers. My model is an extension of search and matching model of the labor market by Pissarides (2000) that captures social networks in the matching process and allows for heterogeneity in the job loss rates of the workers.

Firstly, I take the positive role of social networks as helping members of the network to find jobs as given and formalize the idea by introducing a fixed search effort from employed workers to the unemployed members of the network in the matching function. The matching rate of an unemployed

¹See Montgomery (1991), Mortensen and Vishwanath (1994), Calvo-Armengol (2004), Calvo-Armengol and Jackson (2004, 2007), Calvo-Armengol and Zenou (2005), Fontaine (2007) and Cahuc and Fontaine (2007).

worker depends on his search effort and the number of vacancies created by the firms. The benefit of being a member of the social network is a higher arrival rate of job offers resulting from the help of employed workers which depends on the aggregate conditions in the labor market. The larger the network, the more help unemployed members of the network obtain, therefore the matching rate is increasing in the size of the network, but in a larger network the help from employed members shared by a bigger group of unemployed therefore matching rate is concave in the size of the network. This additional effort is costly for the employed workers but there are potential benefits in terms of regaining employment through their social network in the case of a job loss. Employed workers take the labor market conditions and the size of the network as given and decide whether to exert effort for helping unemployed members and the equilibrium size of the social network is determined endogenously by the individual decisions.

Secondly, I characterize the equilibrium social network. I show that there two stationary equilibria one with an empty network and one with a positive measure of workers in the network. The equilibrium with a positive measure of workers has the property that workers with higher probability of a job loss join the social network, which is in line with the empirical observation. The incentive for the employed workers to exert the costly effort for the unemployed workers is the benefit they will derive from higher arrival rate of offer when they are unemployed and workers who loses their job more frequently have a bigger incentive to participate in the network, whereas for the workers with relatively more stable employment relationships this incentive is smaller.

I show that workers who are members of the social network have a shorter duration of unemployment. In the model, network workers have a higher matching rate compared to the workers who are not a member of the social network therefore their unemployment duration which is the inverse of the matching rate is lower relative to non-network workers. Using data from PSID, I show that average unemployment duration is lower for the workers who use friends and relatives to find jobs. The average duration of unemployment for the workers use social networks is 12.3 weeks, where average unemployment duration for the non-network workers is 16.6 weeks which is also consistent with the

previous empirical evidence.²

Recent literature on social networks show that jobs found through social networks result in lower wages. Using the model, I show that in the equilibrium with a positive measure of workers, network workers obtain lower wages compared to the workers who are not members of the network.³ Model predicts that workers in the network get lower wages in equilibrium. In the model, wages are determined according to Nash-bargaining and workers with higher job loss rate have a lower outside option in the bargaining which result in lower bargained wages and there is also a positive effect on the outside option from higher matching rates for network workers. In equilibrium, the gain from higher matching rates is not enough to compensate the effect of higher job loss rates for the network workers. I present evidence from PSID and show that network workers get lower wages, 6.7 percent on average.

My endogenous framework allows me to analyze how social networks are affected by the labor market conditions. I compare two stationary equilibria with positive measure of network workers with high and low levels of productivity. The equilibrium with low level of productivity gives a higher aggregate unemployment and the one with higher productivity has a lower unemployment rate. I compare the equilibrium size of the network across equilibria and show that the one with higher aggregate unemployment rate has a smaller equilibrium network. If the aggregate unemployment is high, then workers have more likely unemployed friends who are less useful in the job search, workers are less likely to invest in their network. In the empirical analysis, I show that a higher level of aggregate unemployment reduce the use of networks.⁴

I also investigate the effects of policy variables on the use of networks in our model. I focus on the effect of unemployment benefits on the equilibrium size of the network. I show that a higher unemployment benefit reduce the equilibrium size of the network. With higher level of unemployment benefits, employed workers have less incentive to join the network. For the empirical evidence, I refer to Bentolli et.al (2008) which use data from U.S. and Europe on the use of networks to find a job and

²See Bentolli et.al. (2008) for an analysis with a different data set.

³See Bentolli et.al (2008) and Topa (2005).

⁴See the data and estimation section for details

in their sample jobs found through social networks are higher in U.S. than Europe with relatively more generous unemployment benefit schemes compared to U.S.

I extend the simple model to the case with many networks. Social networks are mostly limited by geographical distance therefore the number of people, an agent can contact is limited. Areas which are dense in population expose people to more contacts and provide more opportunities for networking, compared to less dense areas. For example, in cities workers are more likely to have bigger networks than in rural areas which provide them with more information on jobs. In a simple extension of the model, I show that in dense areas the equilibrium size of the network is larger compared to less dense areas. Zenou and Wahba (2003) shows that using survey data from Egypt, the probability of obtaining jobs is increasing and strictly concave with population density which they use as a proxy for the transmission of job information and argue that the size of the network can be approximated by the population density of the area.

The rest of the paper is organized as follows. Empirical evidence indicating a positive relationship between job loss rates and the use of networks is provided in the next section. In section 3 the benchmark model is presented. In this section, I describe the environment and then define and characterize the stationary equilibria of the model. Section 4 describes the extension of the simple model to the environment with multiple networks. Lastly, section 5 concludes.

2 Data and Estimation

The data for this project come primarily from Panel Study of Income Dynamics (PSID) which has followed and interviewed a national sample that began with 5000 families. Low income families are oversampled in the original design and weights are developed to account for initial sampling fractions. Our sample covers the survey data from 1991-2007. Our main purpose is to examine the effect of job stability on the use of networks in the labor market and PSID allows us to track the same individual over a period of time and provide information about individuals labor market experience. The time period is chosen not to cover the entire sample period of PSID but this choice enables us to track the same

individual and use of networks by that individual even in this case I end up with an unbalanced panel of observations⁵. Job stability is defined as the number of involuntary job losses for the individual. To determine whether a job loss occurred for this individual, I use the information on job changes in the data. Workers changing jobs or losing jobs without reemployment are asked "What happened with that job? Did the company go out of business, laid off, fired or quit?" Since I am primarily focusing on the involuntary job losses our measure of a job loss does not include job changes due to quits. For the use of networks I examine the survey question "What did you do to find a job" that allows for the following answers: 1) checked with public employment agency 2) checked with private employment agency 3) checked with the current employer 4) checked with other employer directly 5) checked with friends and relatives 6) placed or answered ads. Another limitation associated with the data is that this question answered when the respondent is unemployed, puts a further restriction on the data set. Starting from 1991 I define the network use variable using the answers of the individual to the above specified question, and if the respondent report the use of friends and relatives I count him as a worker using contacts to find a job. In total I have 1225 observations which I track over the sample period. In order to control the effect of personal characteristics other than job loss rates on the use of the network as a job search method I collect data on race, education and age for the individuals in our sample. I also want to compare the average unemployment duration of these two group of workers who used friends and relatives to find a job and who did not. The average unemployment duration for the entire sample is 14.3 weeks. In order to calculate the unemployment duration I focus on the workers who positively respond to the question on the network use in the 1991 survey and focus on the unemployment spells experienced by those individuals and calculate the average length of unemployment spells for the individual. Workers who report use of networks have a shorter duration of unemployment. The average duration of unemployment is 12.3 weeks for the group who report a use of networks and 16.6 weeks for those who reported no use of contacts. I also compiled data on

⁵Our sample period consists of 16 years. I primarily focus on the network use variable. For job losses workers vary according to their attachments to the labor market. I track individuals also out of sample to get a full picture on job losses and remove the effect of years of labor market experience by a separate regression

the wages of the individuals to see the effect of the use of friends and relatives on the wages. Wages are lower for the workers who reported the use of networks compared to other workers, 6.7 percent controlling for individual characteristics. Table 1 reports the summary statistics for the sample.

Using the panel dimension of the data allows us to control for unobserved individual heterogeneity. For the sample I estimated the following regression

$$N_{i,t} = \gamma Jobloss + \beta X_{i,t} + \alpha_i + \epsilon_{i,t} \quad (1)$$

The index i refers to the worker and t refers to the year. The dependent variable is a binary choice variable, it takes the value 1 if network use is reported and 0 otherwise. The dependent variable includes the responses of the workers who are unemployed at the survey year, in the case where the worker is employed I do not have the value for the particular worker which leads us to work with an unbalanced panel of observations. $X_{i,t}$ is a vector of control variables that includes education, race, age for each individual in the sample and aggregate unemployment rate⁶. The regression model is estimated using a panel-logit model with random effects. One advantage of the random effects model is that I can get estimates for time-invariant regressors. Results from the regression are summarized in Table 2.

The key result from our regression is that there is a positive correlation between job loss rates and the use of networks. Specifically, if the individual has experienced one more job loss the probability of using friends and relatives goes up by 0.035. Workers with higher job loss rates are more likely to rely on their social networks to find a job. I also obtain estimates for the effects of education, race and aggregate unemployment rate on the use of networks. More educated workers are more likely to use their network to find jobs. Whites are less likely to use networks compared to blacks and if the aggregate unemployment rate is high workers rely less on their networks to find jobs.

⁶I obtain data on the aggregate unemployment rate from BLS for the sample period and included in the regression to control the effect of total unemployment on the use of networks.

3 Model

This section outline and discuss the basic model which is an extension of Pissarides⁷ search and matching model of labor market. For the sake of simplicity, I first describe the model with a large and unique network. I choose a continuous time formulation in order to simplify some of the derivations.

There is a measure one of workers indexed by i . Workers are infinitely-lived and have a linear utility over consumption of a homogenous good and discount future at a constant rate $r > 0$ meaning that to the extent of uncertainty, workers are risk-neutral⁸. Workers are identical except the exogenous job loss rate λ drawn from the distribution $G(\lambda)$.

Workers are either employed or unemployed at any point in time. An employed worker earns a wage income w , but cannot search. Unemployed workers search for jobs and obtain unemployment benefit $b > 0$ ⁹. The number of firms is potentially infinite. Firms are equally productive and discount future at the same rate as the workers. A firm-worker pair produces p units of output per unit of time. I assume that value of output exceeds the value of not working for the worker, $p > b > 0$. Firms open vacancies by paying a flow cost c and there is no cost for the firm to enter the labor market.

The search in the labor market is characterized by a matching function which is a reduced form representation of labor market frictions¹⁰. Workers in the labor market are either a member of the network or not. I assume that a fraction μ , an endogenous variable to be determined in equilibrium, of workers are embedded in the network. I consider the role of the social networks as a way to help unemployed workers by passing information about the job opportunities in the labor market, I assume that employed network workers do not search for themselves but they search for the unemployed network workers by exerting a fixed search effort $s_n < 1$. Unemployed workers of both types search

⁷The main reference is Pissarides(1985). Pissarides(2000) includes a survey of matching models. For a more recent survey see Rogerson, Shimer and Wright(2005).

⁸Therefore workers expected present value of utility is simply the expected present value of income. I can alternatively assume that workers are risk-averse but there is complete insurance against idiosyncratic income risk. In this case, it would also be optimal for workers to maximize the expected present value of income.

⁹Note that unemployment benefits do not serve an insurance role in this environment.

¹⁰See Petrongolo and Pissarides(2001) for a survey of the literature on the matching functions.

with a unit intensity. Aggregate search effort in the market, denoted by S is given by:

$$S = u_n + (\mu - u_n)s_n + u_{nn} \quad (2)$$

where u_n is the measure of unemployed workers in the network, u_{nn} the measure of the non-network workers and μ is the total measure of network workers.

This formulation specifies the rate at which new matches are created in the labor market by a time-invariant matching function:

$$m = M(S, v) \quad (3)$$

where v is the measure of firms with an open position. I assume that the matching function is increasing and strictly concave in each argument and constant returns to scale in both arguments and satisfies Inada conditions. The rate at which a job offer reaches the labor market is given by:

$$q(\theta) = \frac{M(S, v)}{S} = M(\theta) \quad (4)$$

where $\theta = v/S$ is the labor market tightness. Since the mass of job-seekers includes both network and non-network workers, arrival rates for these two groups are determined by the associated shares in the aggregate search effort S .

The effective matching rate $q_n(\theta)$ for a worker in the network reads:

$$q_n(\theta) = \left[1 + s_n \frac{\mu - u_n}{u_n} \right] q(\theta) \quad (5)$$

Similarly, the rate at which a worker outside the network meets a firm, $q_{nn}(\theta)$ is equal to $q(\theta)$. The rate at which a vacant firm meets a worker is given by:

$$\rho(\theta) = \frac{M(S, V)}{V} = \frac{q(\theta)}{\theta} \quad (6)$$

Again, noting that firms are more likely to meet network workers, the associated rate is equal to $\rho_n(\theta) = q_n(\theta)/\theta$ for the network workers and $\rho_{nn}(\theta) = q_{nn}(\theta)/\theta$ for the non-network workers. The properties of matching function implies the matching rate of workers (firms) is increasing (decreasing) in θ and I assume that $\lim_{\theta \rightarrow 0} q(\theta) = \lim_{\theta \rightarrow \infty} \theta q(\theta) = \infty$ and $\lim_{\theta \rightarrow \infty} q(\theta) = \lim_{\theta \rightarrow 0} \theta q(\theta) = 0$

With matching frictions, both workers and firms have some bargaining power, below I will determine wages according to Nash-bargaining solution. I will consider the stationary equilibria where aggregate variables are constant over time.

3.1 Values

In this section, I describe the environment through Bellman equations.

I assume that workers in the network exert a fixed search effort $s_n < 1$ for the unemployed network workers, which can be thought as the investment in the network and the cost of this effort is denoted by k ¹¹ The net present values of an unemployed worker in the network, U_i^n , and of a matched worker, E_i^n , are given by:

$$rU_i^n = b + q_n(\theta)[E_i^n - U_i^n] \quad (7)$$

$$rE_i^n = \omega - k - \lambda_i[E_i^n - U_i^n] \quad (8)$$

The first equation gives the flow value of an unemployed network worker which is the sum of the value of not working, b , and the matching probability times the capital gain upon getting a job

¹² An employed network worker gets a wage w , pays the cost of effort s_n , but the job ends at the individual rate λ_i , leaving the worker unemployed. Note that I assume that search intensities are

¹¹I assume a constant cost k for the sake of simplicity and because of the fact that effort is constant for the network. The main result of the paper can be extended to a case where the cost is convex in the size of the network.

¹²This equation is written in the flow form but it can be derived from a discrete time. Suppose that value of being unemployed is constant from the perspective of an employed network worker and look at a period of length Δ . During the period Δ employed network workers get a wage w and cost of the effort for the unemployed network workers is incurred. At the end of the period match is lost with probability $\lambda_i \Delta$ and continues with probability $1 - \lambda_i \Delta$. Therefore $E_i^n(t) = w - k + (1 - \lambda_i \Delta)e^{-r\Delta}E_i^n(t + \Delta) + \lambda_i \Delta e^{-r\Delta}U_i^n$. Subtracting $e^{-r\Delta}E_i^n(t + \Delta)$ from both sides and dividing by Δ we obtain $\frac{E_i^n(t) - E_i^n(t + \Delta)}{\Delta} + \frac{1 - e^{-r\Delta}}{\Delta}E_i^n(t + \Delta) = w - k - \lambda_i e^{-r\Delta}(E_i^n(t + \Delta) - U_i^n)$. Taking limit as $\Delta \rightarrow 0$. The left hand side becomes $\dot{E}_i^n(t) + rE_i^n$ and since $\dot{E}_i^n(t)$ is equal to zero in a stationary equilibrium, we get the flow equation.

exogenous. Unemployed workers do not choose their intensity of search and being in the network requires exerting s_n . The basic mechanism is that a cost is paid when employed with expected future benefit from higher arrival rate offers.

The net present values for the non-network workers are:

$$rE_i^{nn} = \omega - \lambda_i[E_i^{nn} - U_i^{nn}] \quad (9)$$

$$rU_i^{nn} = b + q_{nn}(\theta)[E_i^{nn} - U_i^{nn}] \quad (10)$$

where non-network workers do not exert s_n when employed, but they do not benefit from the network in the case of unemployment.

Network decision: Taking the labor market conditions and the size of the network as given, employed workers decide whether to exert the effort s_n by considering the trade-off between the cost of being in the network to the potential benefit of regaining employment through the network in the future, therefore at any instant of time, employed workers compare the value of being in the network, E_i^n to the value of being outside of the network, E_i^{nn} . Employed workers who have chosen to be outside of the network do not obtain any benefit from the network in the case of unemployment. Observe that due to the stationarity of the problem, an employed worker who has chosen to be in the network will always choose to be in the network in an employment spell. Therefore, the decision problem of the employed workers can be written as:

$$\max_{join, leave} (E_i^n, E_i^{nn}) \quad (11)$$

The above decision problem will give a cut-off value λ^* , and the size of the network is obtained using the distribution of job loss rates, $G(\lambda)$.

Denote the net present value of a matched firm with a worker type i as J_i . Given the productivity

of the match p and the wage rate w , J_i satisfies:

$$rJ_i = p - w - \lambda_i(J_i - V) \quad (12)$$

where V is the value of the firm when unmatched. For a matched firm, the flow value of being matched with a worker type i is equal to the flow profit $p - w$ minus the expected capital loss resulting from a separation of the match. The value of a vacant firm is given by:

$$rV = -c + \rho_n(\theta) \int_{\{\lambda_i: i \in Network\}} [J - V] dG(\lambda) + \rho_{nn}(\theta) \int_{\{\lambda_i: i \notin Network\}} [J - V] dG(\lambda) \quad (13)$$

Observe that there is a flow cost of posting a vacancy and an expected capital gain from the chance of meeting a worker, with different rates for network and non-network workers.

Wages are bargained by the firm and the worker after they meet and firm has perfect information about the type of the worker. All firms are identical and any firm-worker pair produces p . Wages can be renegotiated at each instant of time without any cost. I denote the share parameter for the workers in the Nash-bargaining as β . Wages satisfy the standard Nash bargaining solution:

$$(1 - \beta) [E_i^j - U_i^j] = \beta [J_i - V] \quad (14)$$

where $j = n, nn$.

There is an infinite supply of firms that can post vacancies and there is no cost of entry. Therefore, in equilibrium value of a posted vacancy is zero, $V = 0$. The free entry condition together with the definition of the value of posted vacancy (12) implies:

$$c = \rho_n(\theta) \int_{\{\lambda_i: i \in Network\}} J_i dG(\lambda) + \rho_{nn}(\theta) \int_{\{\lambda_i: i \notin Network\}} J_i dG(\lambda) \quad (15)$$

In the stationary equilibrium, the inflows and outflows from unemployment will be equal for both

group of workers. For network workers stationary unemployment rate is given by:

$$\int_{\{\lambda_i: i \in \text{Network}\}} \lambda(\mu - u_n) dG(\lambda) = q_n(\theta)u_n \quad (16)$$

similarly, for the non-network workers:

$$\int_{\{\lambda_i: i \notin \text{Network}\}} \lambda(1 - \mu - u_{nn}) dG(\lambda) = q_{nn}(\theta)u_{nn} \quad (17)$$

Therefore, in a stationary equilibrium the inflow to unemployment, job loss rate times the measure of employed equals to the outflow from unemployment, job finding rate times the measure of unemployed for both the network and non-network workers.

Definition 1 *A stationary equilibrium for this economy is a set*

$$(E_i^{j*}, U_i^{j*}, J_i^*, w_i^{j*}, u_n^*, u_{nn}^*, \theta^*, \mu^*)$$

consistent with the following conditions given k and s_n for all i and $j = n, nn$:

- (i) *Wages satisfy the Nash-bargaining solution given by (13)*
- (ii) *The unemployment rate for each group is given by (15)-(16)*
- (iii) *The equilibrium measure of the firms is determined by the free entry condition given by (14)*
- (iv) *The size of the network is determined by the individual decisions described by (10)*
- (v) *The value functions for the workers and firms satisfy (6)-(7)-(8)-(9)-(11) and (12)*

3.2 Characterization of Equilibrium

To characterize the equilibria, I first substitute the value functions of employed workers and matched firms into the Nash-bargaining solution which implies:

$$\frac{E_i^n - U_i^n}{\beta} = \frac{J_i}{1 - \beta} = \frac{p - rU_i^n - k}{r + \lambda_i} \quad (18)$$

and

$$\frac{E_i^{nn} - U_i^{nn}}{\beta} = \frac{J_i}{1 - \beta} = \frac{p - U_i^{nn}}{r + \lambda_i} \quad (19)$$

Substituting $E_i - U_i$ from (17) and (18) into (6) and (9) I obtain:

$$rU_i^n = \frac{q_n(\theta)(p - k)}{r + \lambda_i + q_n(\theta)\beta} \quad (20)$$

$$rU_i^{nn} = \frac{q_{nn}(\theta)p}{r + \lambda_i + q_{nn}(\theta)\beta} \quad (21)$$

where I express the values of unemployed workers in terms of the arrival rates and the size of the network μ and parameters of the model.

Using (19) and (20) I can write the free entry condition as:

$$c = \frac{q_n(\theta)}{\theta}(1 - \beta) \int_{i \in \text{Network}} \frac{(r + \lambda)(p - k)}{r + \lambda + q_n(\theta)} dG(\lambda) + \frac{q_{nn}(\theta)}{\theta}(1 - \beta) \int_{i \notin \text{Network}} \frac{(r + \lambda)p}{r + \lambda + q_{nn}(\theta)} dG(\lambda) \quad (22)$$

Observe that using the properties of the matching function I can show that given μ right-hand side of the equation is monotonically decreasing in θ which allows us uniquely determine the labor market tightness and with the knowledge of θ I can solve for the stationary unemployment rate using the equations (15) and (16). Taking the labor market equilibrium and the size of the network as given, employed workers decide to join or leave the network by comparing the equilibrium values E_i^n and E_i^{nn} .

The equilibria of the model in terms of aggregate variables $(\theta^*, \mu^*, u_n^*, u_{nn}^*)$ are completely characterized by the following equations:

$$c = \frac{q_n(\theta)}{\theta}(1 - \beta) \int_{i \in \text{Network}} \frac{(r + \lambda)(p - k)}{r + \lambda + q_n(\theta)} dG(\lambda) + \frac{q_{nn}(\theta)}{\theta}(1 - \beta) \int_{i \notin \text{Network}} \frac{(r + \lambda)p}{r + \lambda + q_{nn}(\theta)} dG(\lambda) \quad (23)$$

$$\int_{\{\lambda_i: i \in \text{Network}\}} \lambda(\mu - u_n) dG(\lambda) = q_n(\theta)u_n \quad (24)$$

$$\int_{\{\lambda_i: i \notin \text{Network}\}} \lambda(1 - \mu - u_{nn}) dG(\lambda) = q_{nn}(\theta)u_{nn} \quad (25)$$

$$\lambda^* \text{ obtained from } \max_{\text{join, leave}} (E_i^n, E_i^{nn}) \quad (26)$$

The first equation is obtained using the value functions of the workers and the firms, Nash bargaining and free entry condition, which determines the equilibrium number of firms in the labor market. To solve for the equilibria, I proceed in two steps. First, taking the size of the network, μ as given, I characterize the equilibrium in the labor market. Then, in a second moment, I find workers' optimal decisions which are consistent with the labor market equilibrium.

Taking the size of the network, μ , as given labor market resembles to the one presented by Pissarides(2000) and the equilibrium is completely characterized by equations (22), (23) and (24). Given μ , because the matching function exhibits constant returns to scale and satisfies Inada conditions, right-hand side of the expression in (22) is monotonically decreasing in the labor market tightness θ and left-hand side is constant thus θ is uniquely determined by (22). Using the knowledge of θ , unemployment rates for the two groups can be solved in a second step using stationary unemployment equations. As I show in the appendix, the difference $E_i^n - E_i^{nn}$ is monotonically increasing in λ_i which allows us uniquely determine the threshold level λ^* at which employed workers are indifferent between being in the network and being outside of the network¹³. Once the aggregate variables are determined, I can solve for the values and the wages using the value functions described in the equilibrium definition and completely characterize the solution of the model.

Therefore, equations (22)-(24) completely characterize the equilibria of the model and the existence and multiplicity of equilibria can be investigated through the analysis of the existence and multiplicity of $(\theta^*, \mu^*, u_n^*, u_{nn}^*)$. Our main finding is stated below and proven formally in the appendix.

Proposition 1 *If meeting technology displays constant returns to scale, then there are two stationary*

¹³Without loss of generality I assume that indifferent worker joins the network

equilibria, one with an empty network, $\mu^ = 0$ and one with a positive measure of workers in the network, $\mu^* > 0$ and the equilibrium with a positive measure of network workers has the property that workers with higher rate of job loss join the network, i.e. they choose to join the network if $\lambda > \lambda^*$ and leave otherwise and $\mu^* = 1 - G(\lambda^*)$.*

First observe that empty network is always an equilibrium. There is also an equilibrium with a positive measure of workers in in the network. The benefit of being in the network is increasing and strictly concave in the size of the network and determined by the equilibrium market tightness θ and because the cost of being in the network is linear in the size, I determine equilibrium network size as the point where marginal benefit equalizes marginal cost which is unique.

The larger the network, there are more employed workers in the network which provide more information to the unemployed ones which increases the chance to find a job through the network. More formally, fixing the number of firms, increasing the size of the network μ will create more search effort on the network side and increase the arrival rate. On the other hand, if the size of the network gets larger, the search effort made by the employed workers is shared by a larger group of unemployed workers and higher search effort from the network creates congestion effect. Note that in our model this result follows from the CRS property of the aggregate matching function.

The relation between probability of finding a job through social network and the size of the network is studied by Zenou and Wahba (2003). They provide a model to account for this relation without explicitly modeling the matching in the labor market, and test the predictions using survey data from Egypt and they found that in relatively dense areas probability of finding job using social contacts is higher and they also show that when the area becomes too dense the probability increases at a decreasing rate. They argue that people are more likely to form contacts in denser areas, a point which I will study in more detail in the model with multiple networks.

Proposition 2 *Network workers get lower wages and they have shorter duration of unemployment compared to non-network workers.*

Workers who use the social contacts in our model are the workers who have a disadvantage in terms of higher job loss rates and therefore they have lower outside options in the market. I think social networks as a mechanism in which people help each other to regain employment in the case of a job loss and increases their outside option in the bargaining with the firms. Consider a worker with a higher job loss rate λ , then in the bargaining with the firm, value of the match is lower for the firm and worker is offered a lower wage.

More formally, I compare the wage equations for network and non-network workers. Equilibrium wages are given by the following equations¹⁴:

$$w_i^n = \beta p + (1 - \beta)(rU_i^n - k) \quad (27)$$

$$w_i^{nn} = \beta p + (1 - \beta)rU_i^{nn} \quad (28)$$

for network and non-network workers respectively. Thus, wages are weighted averages of productivity and the flow value of unemployment. Observe that for the network workers flow value of unemployment is given by $rU_i^n - k$ which includes benefit from the network in terms of higher arrival rates and the flow cost of effort exerted when employed. The equilibrium with positive measure of workers in the network is characterized by a threshold job loss rate where workers with job loss rates above this threshold participate in the network and do not participate otherwise. Equations (19) and (20) shows that the flow value of being unemployed is monotonically decreasing in λ .

Take any two workers with $\lambda_i < \lambda^* < \lambda_j$. Then in stationary equilibrium worker j chooses to be in the network and worker i will be outside of the network. Comparing the wages, the wage for non-network workers is greater than the wage of network worker if and only if $(1 - \beta) [r(U_i^{nn} - U_j^n) + k] >$

¹⁴See Appendix for derivations.

0.

Because flow values are decreasing in λ I have $U_i^n > U_j^n$. By the revealed choices of workers in equilibrium, worker i has chosen to be out of the network thus $U_i^{nn} > U_i^n$. Combining with the inequality above gives $U_i^{nn} > U_j^n$ for all i and j such that $\lambda_i < \lambda^* < \lambda_j$. Therefore the wages of non-network workers will be higher in equilibrium.

I assume that the type of the worker is known by the firm. This may seem a restrictive assumption but firms can obtain information about the labor market experience of a particular worker from many sources, previous employers etc. and workers who have changed many jobs can be seen as less stable matches by the firm and result in a lower wage offers and therefore the effect can be persistent on the worker side. Given this situation for the worker, he is more likely to invest in social networks to secure employment in the case of a job loss.

Shorter unemployment duration for the workers who are using their social contacts to get job, is a direct consequence of higher job arrival rates for this group. Given the aggregate labor market tightness θ , network workers have a higher rate of arrival of offers because of the additional effort coming from the employed workers in the network, therefore their hazard rate is lower which is the inverse of the arrival rates.

Recently, Bentolli et.al (2008) have studied this question both theoretically and empirically. They find using data from US and Europe that social contacts reduce unemployment duration by 1-3 months on average and they are associated with wage discounts of at least 2.5 percent. Previous theoretical research on the role of the networks predict that jobs found through contacts pay higher wages and their paper provides a static model which is consistent with these observations. The model presented in this paper can be seen as an another attempt to explain the observed data facts and as a complementary explanation which obtain the result using a dynamic model which takes into account the aggregate inflows and outflows into unemployment and their effects on the labor market outcomes. The mechanism in our model simply takes into account the benefits and the costs of the networks and endogenously

determine the workers who use social networks in equilibrium and the characteristics of the workers using contacts are the main factor which is affecting their labor market outcomes. As I have showed, in equilibrium the more disadvantaged workers are using the social contacts more intensively and they have lower outside options which result in a relatively worse outcomes in bargaining.

Our endogenous framework allows us to understand the effects of aggregate labor market conditions on the the social networks. For this purpose, I have compared two stationary equilibria (with a positive measure of network workers) of our model, one with high and one with low level of productivity where I have low and high levels of aggregate unemployment rates, respectively¹⁵. When the aggregate unemployment rate is high, then workers have more likely unemployed friends who are less useful in the job search and because there is a cost of maintaining links to the social network, workers are less likely to invest in their network and because the size of the network is getting smaller, the probability of obtaining a job through the network is lower.

More formally, with a lower level of productivity, there are less active firms in equilibrium therefore given the search effort on the worker side, labor market tightness goes down, which decreases the benefit of the network. Because the cost of being in the network k is independent of aggregate conditions, the equilibrium size of the network decreases thus the probability to find a job through the network goes down as well. Following proposition summarizes the effect of a decrease in productivity on the social network.

Proposition 3 *Assume two different productivity levels p_l and p_h such that $p_h > p_l$ and the equilibrium sizes of network are μ_h and μ_l , then $\mu_h > \mu_l$.*

More formally, with a lower level of productivity, there are less active firms in equilibrium therefore given the search effort on the worker side, labor market tightness goes down, which decreases the benefit of the network. Because the cost of being in the network $k(\mu)$ is independent of aggregate conditions, the equilibrium size of the network decreases thus the probability to find a job through the network goes down as well.

¹⁵For example consider a decrease in p which result in an equilibrium with higher aggregate unemployment

Our model also allows us to analyze the effects of policy variables on the use of social networks. I will focus on the unemployment benefits. It has been argued by previous research that unemployment benefits create a disincentive effect for the job search and induce longer unemployment durations. The following proposition summarizes the relationship between unemployment benefits and the use of social networks.

Proposition 4 *A more generous unemployment benefit scheme will reduce the use of social network, i.e. the higher the unemployment benefit b , the lower the size of the network μ .*

When the unemployment benefit b is higher, workers have less incentive to invest in their social network. More formally, higher benefits increase the value of an unemployed worker, which is the outside option of the worker and increases the threshold value λ^* , hence reducing the size of the network.

4 Model with Multiple Networks

The model with multiple networks is a natural extension of the model with a large, unique network. Results similar to the previous section can be obtained by slight modifications. Social networks are mostly limited by geographical distance therefore the number of people an agent can contact is limited. Areas which are dense in population expose people to more contacts and provide more opportunities for networking, compared to less dense areas. For example, in cities workers are more likely to have bigger networks than in rural areas which provide them with more information on jobs. I will formalize this idea by a simple extension of the presented model.

Suppose that each worker i is randomly assigned to a partition F_j and draw λ from $G(\lambda)$. The set F_j can be thought as the set of acquaintances of the individual i from which he chooses his network. I assume that there are N such partitions, i.e. $\{F_1, F_2, \dots, F_N\}$. The network decision is now restricted to the partition F_j for the worker i .

Workers in the labor market are either a member of the network or not. I again assume that employed network workers help unemployed network workers by exerting a fixed search effort $s_n < 1$. I assume that s_n is the same for the networks in each partition. I denote the size of the network in partition F_j as μ_j following the notation introduced in the previous section.

The matching in the labor market is characterized by a matching function $M(S, v)$ where S is the aggregate search effort and v is the measure of vacancies. Aggregate search effort S is given by:

$$S = \sum_{j=1}^N u_{j,nn} + \sum_{j=1}^N u_{j,n} + \sum_{j=1}^N s_n(\mu_j - u_{j,n}) \quad (29)$$

where $u_{j,nn}$ is the measure of unemployed non-network workers, $u_{j,n}$ is the measure of unemployed network workers and μ_j is the measure of network workers in partition j .

The rate at which a job offer reaches the labor market is given by:

$$q(\theta) = \frac{M(S, v)}{S} = M(\theta) \quad (30)$$

where $\theta = v/S$ is the labor market tightness.

The effective matching rate $q_{j,n}(\theta)$ for a worker in the network in F_j is given by:

$$q_{j,n}(\theta) = \left[1 + s_n \frac{\mu_j - u_{j,n}}{u_{j,n}} \right] q(\theta) \quad (31)$$

and the rate at which a worker who is not a member of a network $q_{j,nn}(\theta)$ is equal to $q(\theta)$. The rate at which a firm meets a network worker from F_j reads $\rho_{j,n}(\theta) = q_{j,n}(\theta)/\theta$ and for the non-network workers $\rho_{j,nn}(\theta) = q_{j,nn}(\theta)/\theta$.

Value functions are similar to one network case. The net present values of an unemployed worker in the network in partition j , $U_{i,j}^n$, and of a matched worker, $E_{i,j}^n$, are given by:

$$rU_{i,j}^n = b + q_{j,n}(\theta)[E_{i,j}^n - U_{i,j}^n] \quad (32)$$

$$rE_{i,j}^n = \omega - k - \lambda_i[E_{i,j}^n - U_{i,j}^n] \quad (33)$$

Values for the non-network workers are:

$$rE_{i,j}^{nn} = \omega - \lambda_i[E_{i,j}^{nn} - U_{i,j}^{nn}] \quad (34)$$

$$rU_{i,j}^{nn} = b + q_{j,nn}(\theta)[E_{i,j}^{nn} - U_{i,j}^{nn}] \quad (35)$$

Network decision: Employed workers take the labor market conditions and the size of the network in his partition as given and decide whether to exert the effort s_n , which is the same for all partitions, by considering the cost and benefit of joining the network. With multiple networks, the network decision is restricted to the partitions and an employed worker who has chosen to be outside of the network do not get any help from the network. The decision problem of worker i in partition j is given by:

$$\max_{join, no, leave} (E_{i,j}^n, E_{i,j}^{nn}) \quad (36)$$

The net present value of a matched firm is denoted by J_i . Given the productivity of the match p and the wage rate w , J_i satisfies:

$$rJ_i = p - \omega - \lambda_i(J_i - V) \quad (37)$$

where V is the value of the firm when unmatched. For a matched firm, the flow value of being matched with a worker type i is equal to the flow profit $p - w$ minus the expected capital loss resulting from a separation of the match.

The value of a vacant firm satisfies:

$$rV = -c + \rho_n(\theta) \int_{\{\lambda_i: i \in Network\}} [J - V] dG(\lambda) + \rho_{nn}(\theta) \int_{\{\lambda_i: i \notin Network\}} [J - V] dG(\lambda) \quad (38)$$

In a stationary equilibrium, the inflows and outflows from unemployment are equal for both groups

of workers in each partition I defined. For network workers in partition F_j the stationary unemployment is given by:

$$\int_{\{\lambda_i: i \in \text{Network}\}} \lambda(\mu_j - u_{j,n}) dG(\lambda|F_j) = q_{j,n}(\theta)u_{j,n} \quad (39)$$

where $G(\lambda|F_j)$ is the conditional distribution of job loss probabilities in partition F_j . Similarly for non-network workers, in a stationary equilibrium unemployment satisfies:

$$\int_{\{\lambda_i: i \notin \text{Network}\}} \lambda(F_j - \mu_j - u_{j,nn}) dG(\lambda|F_j) = q_{j,nn}(\theta)u_{j,nn} \quad (40)$$

Wages are bargained by the firm and the worker after they meet and I assume that firm has perfect information about the type of the worker. Again denoting the share parameter for the worker in the bargaining as β , wages satisfy the Nash-bargaining solution:

$$(1 - \beta) [E_{i,j}^n - U_{i,j}^n] = \beta [J_i - V] \quad (41)$$

for the network workers and

$$(1 - \beta) [E_{i,j}^{nn} - U_{i,j}^{nn}] = \beta [J_i - V] \quad (42)$$

for non-network workers.

Definition 2 *A stationary equilibrium for this economy is a set*

$$(E_{i,j}^n, U_{i,j}^n, E_{i,j}^{nn}, U_{i,j}^{nn}, J_i, w_i^j, u_{j,n}, u_{j,nn}, \theta, \mu_j)$$

consistent with the following conditions given k and s_n for all i and j :

- (i) *Wages satisfy the Nash-bargaining solution given by (40) and (41)*
- (ii) *The unemployment rate for each group is given by (38)-(39)*
- (iii) *The equilibrium measure of the firms is determined by the free entry condition given by (37)*
- (iv) *The size of the network is determined by the individual decisions*

described by (35) (v) The value functions for the workers and firms satisfy (31)-(32)-(33)-(34)-(36).

4.1 Characterization of Equilibrium

The characterization of equilibria is similar to the model described in the previous section. For completeness, I reproduce the equations extended for the multiple network case. Substituting the value functions into the Nash-bargaining solution I obtain:

$$\frac{E_{i,j}^n - U_{i,j}^n}{\beta} = \frac{J_i}{1 - \beta} = \frac{p - rU_{i,j}^n - k}{r + \lambda_i} \quad (43)$$

and

$$\frac{E_{i,j}^{nn} - U_{i,j}^{nn}}{\beta} = \frac{J_i}{1 - \beta} = \frac{p - U_{i,j}^{nn}}{r + \lambda_i} \quad (44)$$

Using the difference between the values of employed and unemployed workers I get flow value of unemployment for network and non-network workers in partition F_j .

$$rU_{i,j}^n = \frac{q_{j,n}(\theta)(p - k)}{r + \lambda_i + q_{j,n}(\theta)\beta} \quad (45)$$

$$rU_{i,j}^{nn} = \frac{q_{j,nn}(\theta)p}{r + \lambda_i + q_{j,nn}(\theta)\beta} \quad (46)$$

Combining these equation with the free entry condition I obtain:

$$c = \sum_{j=1}^N \frac{q_{j,n}(\theta)}{\theta} (1 - \beta) \int_{i \in \text{Network}} \frac{(r + \lambda)(p - k)}{r + \lambda + q_n(\theta)} dG(\lambda|F_j) +$$

$$\sum_{j=1}^N \frac{q_{j,nn}(\theta)}{\theta} (1 - \beta) \int_{i \notin \text{Network}} \frac{(r + \lambda)p}{r + \lambda + q_{nn}(\theta)} dG(\lambda|F_j)$$

By the properties of the matching function, given the composition of the networks in the labor

market, right-hand side of the free entry condition is monotonically increasing in θ and left-hand side is independent of θ , therefore I can uniquely determine the equilibrium labor market tightness. Given the labor market tightness and the sizes of the networks in each partition F_j , I can solve for stationary unemployment in each partition for network and non-network workers uniquely. Taking the labor market conditions and the network in each partition F_j as given employed workers decide to participate in the network or not by comparing the equilibrium values.

Again, we characterize the equilibria of the model in terms of aggregate variables $(\theta, \mu_j, u_{j,n}, u_{j,nn})_{j=1,\dots,N}$ which satisfy the free entry condition, stationary unemployment for network and non-network workers in each partition F_j and network decision of employed workers.

Observe that the equilibria of the model define a multi-dimensional fixed point problem. The size of the network in partition F_j depends on the sizes of the networks in other partitions. In order to put more structure to the problem I assume that the conditional distribution of job loss rates across partitions are identical, i.e. $G(\lambda|F_j) = G(\lambda|F_l)$ for all j and l .

To solve for the equilibria, I again proceed in two steps. First, taking the networks in each partition as given, I characterize the equilibrium in the labor market. Then, I find the optimal decisions of individuals that are consistent with the labor market equilibrium.

Taking the composition of the network, (μ_1, \dots, μ_N) as given, the equilibrium labor market tightness θ^* is uniquely determined by the free entry condition by using constant returns to scale property of the matching function and Inada conditions. Using the knowledge of θ^* I can solve for stationary unemployment in each partition. For each partition, the difference $E_{i,j}^n - E_{i,j}^{nn}$ is monotonically increasing in λ . Therefore, the equilibrium size of the network in each partition, conditional on the sizes of the networks in other partitions, is either empty or a unique positive measure. The following proposition describes the equilibria in the model with multiple networks.

Proposition 5 *If the matching technology displays constant returns to scale and the conditional distribution of λ is identical across partitions, then there are 2^N stationary equilibria of the model where the size of the equilibrium network in each partition is either empty or one with a positive measure*

of workers in the network. The equilibrium with a positive measure of workers in each partition F_j has the property that workers with higher rate of job loss join the network, i.e. they choose to join the network if $\lambda > \lambda_j^*$ and leave otherwise and $\mu_j^* = 1 - G(\lambda_j^*|F_j)$.

The empty network is always an equilibrium. In each partition, there is also an equilibrium network with a positive measure of workers but in the model with multiple networks the equilibrium size of the network in each partition also depends on the sizes of the networks outside of that partition. The property that still holds for the multiple network model is that the equilibrium network is determined as the point where the benefit obtained from participating in the network, which is an equilibrium variable, is equal to the cost of participating, which is unique given the aggregate conditions in the labor market.

As I argued above, the number of social contacts that an individual has, are limited. Areas which are dense in population expose people to more contacts therefore provide more opportunities for networking which lead to larger networks. Zenou and Wahba(2003) shows that using survey data from Egypt, the probability of obtaining jobs is increasing and strictly concave with population density which they use as a proxy for the transmission of job information and argue that the size of the network can be approximated by the population density of the area. To formalize the idea, I use the model with multiple networks. For simplicity, consider two partitions, F_1 and F_2 .¹⁶ where $F_1 > F_2$ and assume that the conditional distribution of λ is identical i.e. $G(\lambda|F_1) = G(\lambda|F_2)$. I focus on the equilibrium with positive measure of workers in each partition. The following proposition state the result and proved in the appendix.

Proposition 6 *Suppose that there are two partitions F_1 and F_2 such that $F_1 > F_2$ and the conditional distribution of λ across partitions is identical i.e. $G(\lambda|F_1) = G(\lambda|F_2)$. Then $\mu_1 > \mu_2$.*

¹⁶The argument can be extended to more then two partitions.

5 Conclusion

It is well established in the empirical literature that social network play an important role in the labor market. Previous research take the social networks as given and explored the effects of social networks on the labor market outcomes. In this paper, I provide a simple model of network formation in the labor market and evaluated the predictions of the model using data from PSID and document the positive relation between job loss rates and use of networks. Our endogenous framework allows us to analyze the effect of conditions in the labor market on the use of networks.

Our model abstracts from the strategic interactions among agents and this is clearly a limitation of our framework. I focus on the simple model as it is a useful starting point to understand the formation of networks in a dynamic model of the labor market. I believe that modelling the strategic interactions among the agents as a repeated game will give us more insights on the equilibrium network formation.

I assume that search intensities are exogenous. It can be argued that the costs of these formal and informal search channels are different. For some workers, it may be more costly to use formal channels of job search compared search through networks and provide another incentive for network formation. This issue is our research agenda.

6 Appendix

Proof of Proposition 1

Using the generalized Nash-bargaining solution we can solve for the value of a firm matched with a network worker as:

$$J_i^n = (1 - \beta) \frac{p - rU_i^n - k}{r + \lambda_i} \quad (47)$$

and the value of a match with a non-network worker is:

$$J_i^{nn} = (1 - \beta) \frac{p - rU_i^{nn}}{r + \lambda_i} \quad (48)$$

Substituting the values of unemployed network and non network workers

$$rU_i^n = \frac{q_n(\theta)(p-k)}{r + \lambda_i + q_n(\theta)\beta} \quad (49)$$

$$rU_i^{nn} = \frac{q_{nn}(\theta)p}{r + \lambda_i + q_{nn}(\theta)\beta} \quad (50)$$

into J_i^n and J_i^{nn} and putting them into the free entry condition we obtain:

$$c = \frac{q_n(\theta)}{\theta}(1-\beta) \int_{i \in Network} \frac{(r+\lambda)(p-k)}{r+\lambda+q_n(\theta)} dG(\lambda) + \frac{q_{nn}(\theta)}{\theta}(1-\beta) \int_{i \notin Network} \frac{(r+\lambda)p}{r+\lambda+q_{nn}(\theta)} dG(\lambda) \quad (51)$$

Taking the size of the network μ as given observe that LHS of the equation is constant. RHS is monotonically decreasing in θ because matching function satisfies the Inada conditions i.e. $\lim_{\theta \rightarrow 0} q(\theta) = \lim_{\theta \rightarrow \infty} \theta q(\theta) = \infty$ and $\lim_{\theta \rightarrow \infty} q(\theta) = \lim_{\theta \rightarrow 0} \theta q(\theta) = 0$. Thus there is a unique θ which solves the free entry condition given μ . Once we determine θ and taking μ as given, we solve for stationary unemployment using equations (23) and (24). Note that given μ and θ , LHS of both expressions are decreasing in the unemployment and RHS is increasing thus there are unique values of u_n and u_{nn} which satisfy (23) and (24). Therefore given μ , the labor market equilibrium is uniquely characterized by (22), (23) and (24). The optimal size of the network is determined by comparing the values of employed network and non-network workers. First observe that $\mu = 0$ is always a fixed point of the problem, because given $\mu = 0$ no worker will join the network. The difference $E_i^n - E_i^{nn}$ can be solved by using the value functions and given by : $(r + \lambda_i - \beta r)(U_i^n - U_i^{nn})$ which is monotonically increasing in λ_i iff $U_i^n - U_i^{nn}$ is monotonically increasing in λ_i and that difference can be obtained using values of unemployed network and non-network workers. Taking the equilibrium in the labor market as given the difference can be expressed as $((p-k)\gamma) - b - k)(r + \lambda_i) - k(1 + \gamma)\beta q(\theta)$, where $\gamma = s_n \frac{\mu - u_n}{u_n}$, is monotonically increasing in λ_i Therefore, there is a unique value λ^* which makes the difference $E_i^n - E_i^{nn}$ equal to zero and the difference is negative below λ^* and positive above λ^* and equilibrium size of the network is given by $1 - G(\lambda^*)$.

Derivation of Wage Equations

Generalized Nash-bargaining solution implies:

$$\frac{E_i^n - U_i^n}{\beta} = \frac{J_i}{1 - \beta} = \frac{p - rU_i^n - k}{r + \lambda_i} \quad (52)$$

and

$$\frac{E_i^{nn} - U_i^{nn}}{\beta} = \frac{J_i}{1 - \beta} = \frac{p - U_i^{nn}}{r + \lambda_i} \quad (53)$$

Solving for $E_i^n - U_i^n$ and $E_i^{nn} - U_i^{nn}$ we obtain

$$E_i^n - U_i^n = \frac{w - k - U_i^n}{r + \lambda_i} \quad (54)$$

$$E_i^{nn} - U_i^{nn} = \frac{w - k - U_i^{nn}}{r + \lambda_i} \quad (55)$$

Substituting back into the Nash-bargaining solution and solving for w we obtain:

$$w_i^n = \beta p + (1 - \beta)(rU_i^n - k) \quad (56)$$

$$w_i^{nn} = \beta p + (1 - \beta)rU_i^{nn} \quad (57)$$

Proof of Proposition 3 The claim in the proposition is that the equilibrium size of the network is smaller if the aggregate productivity is lower. Focusing on the equilibrium with a positive measure of workers in the network, we want to have $\frac{\partial \mu^*}{\partial p} > 0$ or equivalently $\frac{\partial \lambda^*}{\partial p} < 0$. First observe that from the free entry condition

$$c = \frac{q_n(\theta)}{\theta}(1 - \beta) \int_{i \in \text{Network}} \frac{(r + \lambda)(p - b - k)}{r + \lambda + q_n(\theta)} dG(\lambda) + \frac{q_{nn}(\theta)}{\theta}(1 - \beta) \int_{i \notin \text{Network}} \frac{(r + \lambda)p}{r + \lambda + q_{nn}(\theta)} dG(\lambda) \quad (58)$$

we have $\frac{\partial \theta^*}{\partial p} > 0$ because RHS increases for every value of θ . Then using the unemployment equations

$\frac{\partial u_1}{\partial p} = \frac{\partial u_1}{\partial \theta} \frac{\partial \theta}{\partial p}$ and the first derivative is negative and second derivative is positive hence we have $\frac{\partial u_1}{\partial p} <$

0. we write the equation which determines λ^* in implicit form as $F(\lambda^*) = (r(1 - \beta) + \lambda^*)((p - b - k)\gamma + b - k)(r + \lambda^*) - k(1 + \gamma)\beta q(\theta)) = 0$. Taking the derivative of $F(\lambda^*)$ with respect to p and using $\mu^* = 1 - G(\lambda^*)$ we obtain

$$\frac{\partial \lambda^*}{\partial p} 2BC + (p - k)C^2 A - k\beta A q(\theta) + k\beta(1 + \gamma) \frac{\partial q(\theta)}{\partial p} = 0$$

where $B = (p - k)\gamma - b - k$, $C = (r + \lambda^*)$ and $A = -G'(\lambda^*) \frac{\partial \lambda^*}{\partial p} - \frac{\partial u_1}{\partial p} \mu$. solving for $\frac{\partial \lambda^*}{\partial p}$ we get

$$\frac{\partial \lambda^*}{\partial p} = \frac{-\gamma C^2 - \frac{\partial q(\theta)}{\partial p} k\beta(1 + \gamma) + (p - k)C^2 \frac{\partial u_1}{\partial p} + k\beta q(\theta) \frac{\partial u_1}{\partial p}}{2BC + (p - k)C^2 G'(\lambda^*) + k\beta q(\theta) G'(\lambda^*)}$$

The first term in the nominator is negative because $\gamma > 0$ and $\lambda^* > 0$. The sign of the second term depends on the sign of $q'(\theta)$ which is $\frac{\partial q(\theta)}{\partial \theta} \frac{\partial \theta}{\partial p}$ which is positive since $\frac{\partial \theta}{\partial p} > 0$ by free entry condition and $\frac{\partial q(\theta)}{\partial \theta}$ by the properties of matching function hence second term is negative. The sign of the third and fourth term determined by the sign of $\frac{\partial u_1}{\partial p}$ since the remaining terms are positive hence nominator of the expression is negative. The sign of the denominator of the expression depends on the sign of $G'(\lambda^*)$ assuming that the cumulative distribution function is differentiable then $G'(\lambda^*) > 0$ thus we have $\frac{\partial \lambda^*}{\partial p} < 0$.

Proof of Proposition 4 The proof for this proposition is similar to the one with the change in productivity. The claim is an increase in the unemployment benefit will result in a decrease in the equilibrium size of the network i.e. $\frac{\partial \mu^*}{\partial b} < 0$ or equivalently $\frac{\partial \lambda^*}{\partial b} > 0$.

Writing the equation that determines λ^* in implicit form we obtain

$$F(\lambda^*) = (r(1 - \beta) + \lambda^*)((p - k)\gamma - b - k)(r + \lambda^*) - k(1 + \gamma)\beta q(\theta)) = 0$$

Taking the derivative of the expression with respect to b we get

$$\frac{\partial \lambda^*}{\partial b} 2BC + (p - k - 1)C^2 A - k\beta A q(\theta) + k\beta(1 + \gamma) \frac{\partial q(\theta)}{\partial b} = 0$$

where $B = (p - k)\gamma - b - k$, $C = (r + \lambda^*)$ and $A = -G'(\lambda^*)\frac{\partial \lambda^*}{\partial p} - \frac{\partial u_1}{\partial p}\mu$. Solving for $\frac{\partial \lambda^*}{\partial b}$ we obtain

$$\frac{\partial \lambda^*}{\partial b} = \frac{C^2 - (p - k)C^2\frac{\partial u_1}{\partial b}\mu^* - k\beta q(\theta)\frac{\partial u_1}{\partial b} + k\beta(1 + \gamma) + (1 + \gamma)\frac{\partial q(\theta)}{\partial \theta}\frac{\partial \theta}{\partial b}}{2BC + (p - k - 1)G'(\lambda^*) + k\beta q(\theta)G'(\lambda^*)}$$

First, note that $\frac{\partial \theta}{\partial b} < 0$ from the free entry condition and a decrease in the labor market tightness increases the unemployment. Formally, from stationary unemployment equations, given the size of the network, we have $\frac{\partial u_1}{\partial b} > 0$. The first term in the nominator of the expression is positive. The sign of the second term depends on the sign of $\frac{\partial u_1}{\partial b}$, which is negative, hence second term is also positive and similarly the sign of the third term is also positive. The last term in the nominator is also positive since $\frac{\partial \theta}{\partial b} < 0$ and from the properties of the matching function the arrival rate for the workers is increasing in θ that is $\frac{\partial q(\theta)}{\partial \theta} > 0$. The terms in the denominator are all positive. Assuming that $G(\lambda)$ is differentiable we have then $G'(\lambda^*) > 0$. Thus we have $\frac{\partial \lambda^*}{\partial p} < 0$.

Proof of Proposition 5

First observe that taking the network equilibrium $(\mu_1, \mu_2, \dots, \mu_N)$ as given from the free entry condition:

$$c = \sum_{j=1}^N \frac{q_{j,n}(\theta)}{\theta} (1 - \beta) \int_{i \in \text{Network}} \frac{(r + \lambda)(p - k)}{r + \lambda + q_n(\theta)} dG(\lambda|F_j) + \sum_{j=1}^N \frac{q_{j,nn}(\theta)}{\theta} (1 - \beta) \int_{i \notin \text{Network}} \frac{(r + \lambda)p}{r + \lambda + q_{nn}(\theta)} dG(\lambda|F_j)$$

LHS of this expression is constant. RHS is monotonically decreasing in θ because matching function satisfies the Inada conditions i.e. $\lim_{\theta \rightarrow 0} q(\theta) = \lim_{\theta \rightarrow \infty} \theta q(\theta) = \infty$ and $\lim_{\theta \rightarrow \infty} q(\theta) = \lim_{\theta \rightarrow 0} \theta q(\theta) = 0$. Thus there is a unique θ which solves the free entry condition given $(\mu_1, \mu_2, \dots, \mu_N)$. Give the knowledge of θ and taking the network sizes as given, we determine the equilibrium unemployment for network and non-network workers for each partition using:

$$\int_{\{\lambda_i: i \notin \text{Network}\}} \lambda(F_j - \mu_j - u_{j,nn}) dG(\lambda|F_j) = q_{j,nn}(\theta)u_{j,nn}$$

and

$$\int_{\{\lambda_i: i \in \text{Network}\}} \lambda(\mu_j - u_{j,n}) dG(\lambda|F_j) = q_{j,n}(\theta)u_{j,n}$$

Note that LHS of these expressions are decreasing and RHS is increasing in the unemployment thus unemployment is uniquely determined for each group of workers in each partition, hence we have established that for any composition $(\mu_1, \mu_2, \dots, \mu_N)$ there is a unique equilibrium in the labor market. Note that the equilibrium with an empty network in each partition is always an equilibrium. The equilibrium size of the network is determined by comparing the values of employed network and non-network workers in each partition taking the labor market equilibrium and the networks as given. Observe that the equilibrium in one partition depends on the network sizes in other partitions thus the problem is a multidimensional fixed point problem. The difference between values of being employed for network and non-network worker in each partition is $E_{i,j}^n - E_{i,j}^{nn}$ and it can be expressed as $(r + \lambda_i - \beta r)(U_i^n - U_i^{nn})$ and $(U_i^n - U_i^{nn})$ is equal to $((p - k)\gamma) - b - k)(r + \lambda_i) - k(1 + \gamma)\beta q(\theta)$ where $\gamma = s_n \frac{\mu_j - u_{j,n}}{u_{j,n}}$. Thus the difference $E_{i,j}^n - E_{i,j}^{nn}$ is monotonically increasing in λ_i thus there is unique equilibrium in each partition taking the market conditions as given. Therefore in each partition there are two equilibria one one with empty network and one with a positive measure of workers in the network. The equilibrium size in the positive measure equilibrium also depend on the sizes of the networks in other partitions because θ depends on $(\mu_1, \mu_2, \dots, \mu_N)$. Thus there is unique equilibrium with positive measure of network workers in each partition.

Proof of Proposition 6 Suppose that we have two partitions F_1 and F_2 such that $F_1 > F_2$. We focus on the non-trivial equilibrium where the equilibrium size of the network is positive in each partition i.e. $(\mu_1, \mu_2) > 0$ and since we are assuming that the conditional distribution of λ is identical in both partitions it is sufficient to show that the threshold value λ_1^* is smaller than the threshold value in partition two, λ_2^* The equilibrium size of the network in each partition is determined by the difference

between the values of employed network and non-network workers, i.e. $E_{i,1}^n - E_{i,1}^{nn}$ and $E_{i,2}^n - E_{i,2}^{nn}$ which are given by $(r + \lambda_i - \beta r)(U_{i,1}^n - U_{i,1}^{nn})$ and $(r + \lambda_i - \beta r)(U_{i,2}^n - U_{i,2}^{nn})$ and also note that the values for non-network workers are equal in each partition i.e. $E_{i,1}^{nn} = E_{i,2}^{nn}$ and $U_{i,1}^{nn} = U_{i,2}^{nn}$. The difference between $U_{i,1}^n - U_{i,1}^{nn}$ is equal to $((p - k)\gamma_1) - b - k)(r + \lambda_i) - k(1 + \gamma_1)\beta q(\theta)$ where $\gamma_1 = s_n \frac{\mu_1 - u_{1,n}}{u_{1,n}}$ and for the second partition $U_{i,1}^n - U_{i,1}^{nn}$ is equal to $((p - k)\gamma_2) - b - k)(r + \lambda_i) - k(1 + \gamma_2)\beta q(\theta)$ where $\gamma_2 = s_n \frac{\mu_2 - u_{2,n}}{u_{2,n}}$. Since the option of being outside of the network gives the same value in each partition we compare the value of being in the network for both partitions. For the employed workers that depends on the difference $U_{i,1}^{nn} - U_{i,2}^{nn}$ which in turn depends on the arrival rates for each group. Since the first group has a bigger size, for any value of λ_i the return from network is higher for the first group and because the outside options are equal a lower value of λ^* can be achieved in the larger group thus threshold is lower and the equilibrium size of the network is larger for the larger group.

6.1 Data Appendix

Table 1: Sample Characteristics of the Data

Job Search	Friends-Relatives		Other channels	
	Mean	Std. Deviation	Mean	Std. Deviation
Job loss	3.9	1.8	2.4	1.5
Age	30.5	8.3	32.5	9.4
Unemployment duration	12.3	9.6	16.6	9.9
Years of Education	10.6	2.8	12.5	3.2
White	26.6	15.6	32.2	16.6
Black	33.1	18.4	24.5	14.8
Aggregate unemployment rate	5.38	1.18	-	-
Hourly wage rate	7.8	4.9	8.32	5.1

Note: Numbers are panel averages for the sample period. Aggregate unemployment data is obtained from BLS for the sample period. Unemployment duration is calculated using the reports of individuals on the job search question and taking the average duration of an unemployment spells in the subsequent years. Education is reported as the years of education, we included only individuals who are finished their education and active in the labor market among the respondents to the

job search question. For blacks and whites means are in percentage shares.

Table 2: Network Use and Job Loss

Dependent variable : Network Use	Coefficient	p-value
Job Loss	0.035	0.038
Education	-0.021	0.41
White	0.0025	0.46
Black	0.004	0.028
Unemployment	0.002	0.035
Age	-0.003	0.047
Age-Squared	0.0018	0.067
R- Squared	0.16	
No. Of Observations	10234	

Note: Estimation results from panel-logit estimation with random effects. Coefficients are the calculated marginal effects. Year-dummies are included to the estimation for each survey year.

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