

Economics of Open Source: A Dynamic Approach*

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Abstract

We analyze open source licensing and its effects on firms' decisions whether to use the open source or not and on the incentives for innovation, through a dynamic model of innovation and competition in an environment with a ladder type technology. We model the basic features of the General Public License (GPL), one of the most popular open source licenses and study how firms behave under this license when competition is present. Under the GPL, any innovative findings using open source must also be open source in the subsequent periods. This obligation creates a trade-off between stimulating innovation and promoting disclosure. By using the open source, a firm can increase its technology level and therefore its probability of innovation success and of achieving a greater profit in that period. However, since any innovative findings of the firm, which is using the open source, will be used by the other firms in later periods, firm's expected future revenue may decrease. We

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analyze this trade-off and show that if a firm has the same or higher technology level as the open source, it does not use/join the open source. On the other hand, if a firm has a lower level of technology than the open source, it is optimal to use the open source.

Keywords : open source, General Public License, innovation, research and development, public good, dynamic games

JEL Classification Numbers: L17, D21, C73, O3

1 Introduction

Open software development involves a major deviation from the private investment model of innovation; open source innovators freely share the proprietary software that they have developed at their private expense. For example, Linux, a computer operating system, is evolving with many independent developers revealing the source code to develop and refine it. Its source code is *open* in the sense that anyone has free access to it. One of the most popular web server has always been an open source software. For example, Netcraft's statistics on web servers show that an open source software web server, Apache, has been dominating the public Internet web server market ever since April 1996.¹

The success of open source software raises many questions about innovation policies with non-traditional property rights.² We particularly focus on the fact that although the source code of open source software is freely available, open source programs are distributed under very precise licensing agreements. The broad purpose of this paper is to improve our understanding on how a certain form of license affects firms' incentives of both innovation and participation in an open source community. Specifically, we provide a dynamic model that captures the important characteristics of open source licensing and we explain, to some extent, the open source development phenomenon. To capture the important features of open source innovation and its licensing, we focus on a concrete example, the GNU General Public License (GPL), which is one of the most common licenses.³ GPL has two main features. Firstly, while every user has the freedom to use and modify programs subject to the GPL, such modifications must be distributed under the terms of the license itself, if they are to be distributed at all. Secondly, the GPL does not preclude the commercial exploitation of the software, at any stage. That is, the program users have to maintain the free access to the source, but they do not need to

¹See <http://news.netcraft.com/archives/category/web-server-survey/>

²See Lerner and Tirole (2002) for a broad discussion of issues concerning economics of open source.

³GNU is a recursive acronym which stands for "GNU's Not Unix".

share any profit they make. There are around two hundred Linux open source platform vendors globally and they pool together hundreds of applications, placed on top of an open source operating system, marketed through a number of channels and via a number of different business models. It is evident that there is strong competition in this field. Once open source code is improved by a firm, by its nature, it is accessible to its customers or even to its competitors. However, due to its complexity of programming, the inventor can enjoy advantageous position as the first mover for a span of time.

In this paper, we study these major features of such licensing in open innovation and their effects on incentives for innovation and usage of the open source. In order to capture these key characteristics, namely the dynamic nature of the open source development and the GPL restrictions (successful innovations being subject to GPL) with allowance of open source firms making profit, we use a T-period 3-stage model where, in each period, firms decide whether to use the open source in the first stage, pursue technology advancement through cost-reducing investment in the second stage, and engage in Cournot competition in the third stage. Within the framework of our model, which we believe reflects these important features of the open source innovation in a more direct way than the existing literature does, we study the open source usage decisions of the firms as well as their investment decisions in innovation and competition quantities. We characterize when it is optimal for a firm to join the open source and when it is not, together with the investments in innovation. The main tradeoff in our analysis is stemming from the very nature of the GPL licensing. When a firm is deciding to join the open source community, it will be able to use the technology level of the open source at no direct cost, and this will potentially remove a cost disadvantage in the competition stage against other firms. However, if this firm, after joining the open source community, succeeds in innovation or in cost reduction, then in later periods it has to make it public due to licensing, and thus, might lose a potential cost advantage, which may not be the case if instead it stayed

out of the open source community. This may decrease the expected future revenue of the firm, when joined the open source. We investigate how this trade-off influences firms' open source use decision depending on their technology level relative to the open source technology level. We find that this tradeoff is resolved in a way that when a firm is at the same technology level with (or at a higher level than) the open source, it does not prefer to join the open source. If, however, it is behind the open source in the technology ladder, it optimally chooses to use the open source. Thus, if the open source succeeds consistently in innovation (cost reduction), then the open source will sweep out the proprietary firms, otherwise there will be a set of firms at a higher technology level than the open source.

The literature on economics of open source has been growing since early 2000s. Lerner and Tirole (2002, 2005) provide a general discussion of the economics of open source development and lay out a broad literature review. They point out that the open source developers receive a direct effect in the form of improved open source, since they directly benefit from it, and an indirect effect through signaling their abilities and through reputational gains. They show that the literature mostly considers individual motives, incentives to adopt open source softwares and the effect of competition within an open source environment.⁴ Athey and Ellison (2014) use a dynamic model where the open source user/programmers are motivated by reciprocal altruism. The evolution of the open source depends on the quality and the altruistic developers. Bitzer, Schrettl and Schröder (2007) provide a dynamic model of private provision of a public good and focus on the intrinsic motivation of the programmers to explain the open source development. We are not modeling the firms/developers as altruistic or intrinsically motivated players, rather they are strategic agents who wish to maximize expected profits, as the GPL allows them to do.

Since there is a public good nature in open source innovations, a good amount of

⁴See Lerner, Pathak and Tirole (2006) for an empirical study on the dynamics of contributions to open source software projects.

the literature focuses on the open source development as a public good and uses a static approach. For instance, Johnson (2002) takes a public good approach and uses a static model of private provision of a public good to present welfare results and comparative statics. Modica (2012) considers a two-stage (otherwise static) oligopoly game using a circular city approach and models the open source development from a public good perspective. Atal and Shankar (2014) model the open source development through quality competition in a public good setting. A closely related study is Llanes and Elejalde (2013), where they explore the open source participation decision of n firms, their R&D investment decisions and the prices they pick, using a two-stage game. They characterize the conditions for open-source firms and proprietary firms to coexist. However, their two stage game is played only once, and there is no further dynamics, unlike our current study, where we allow a similar interaction to be repeated over time.

Economides and Katsamakos (2006) study different industry structures, vertically integrated proprietary, vertically disintegrated proprietary, and open source platform with proprietary applications, and show that the profits are highest for the vertically integrated proprietary industry structure. Jaisingh, See-To and Tam (2014) study the effect of existence of an open source software on the behavior of a firm developing a closed source software, on its resource investment and its pricing decision, through a duopolistic framework, where the investment increases the quality and then the price the firm picks determines its demand. Li and Ji (2010) consider a duopoly model where the firms can reduce cost through R&D and compare the welfare effects of price and quantity competition in the presence of technology licensing. They show that Cournot competition results in lower prices, lower industry profit, higher consumer surplus and higher social welfare than Bertrand competition. Mustonen (2005) considers the decision problem of a firm which either chooses to support an existing open source program or not. When the firm supports the open source program, the programs become compatible and there are net-

work effects.⁵ Casadesus-Masanell and Llanes (2011) use a mixed duopoly model where a profit maximizing firm competes with an open source firm, which aims to maximize the value of its open software.⁶ Our model and approach differs from these studies in the way it captures the essence of the GPL licensing in open source innovation environments. Since these studies use a static model, they do not fully capture the main characteristic of the GPL, which is basically “get it for free *now*, pay back *when/if* you succeed.” We believe, our model captures the essence of the GPL in a more direct way: when a firm uses the open source, its successful innovation is made available in later periods, yet in the current period with the innovation success, the firm can enjoy an advantage in the competition stage.

Among the dynamic models studied, Yildirim (2006) considers the free-rider problem when the incentive to induce others to contribute by contributing more is present.⁷ Caulkins et al. (2013) study the question of how long does a firm keep its software proprietary and when does it release it to be open source, over a continuous time dynamic model where firms invest in quality and pick own price for its software and complementary product. Kurt and Zaccour (2011) study a similar problem through a 3 stage duopoly game, where they characterize the conditions under which it is optimal for a firm to open its code. Casadesus-Masanell and Ghemawat (2006), in a dynamic mixed duopoly model, study the competition between an open source firm and a proprietary firm, when there is demand-side learning. They show that the proprietary firm stays in the market even if there is an unbounded learning in the open source side.⁸ Our work differs from these

⁵He characterizes the conditions under which supporting is optimal and also shows that a larger open source programmers’ community does not necessarily increase the welfare.

⁶Assuming the product packages are composed of a base module, an extension module and a complementary service, they consider pure and mixed business models. Under compatibility and incompatibility, they solve for the set of optimal business models.

⁷For more on dynamic voluntary contribution games see, for instance, Admati and Perry (1991) and Marx and Matthews (2000).

⁸For more on duopolistic competition related studies, see also Haruvy, Sethi and Zhou (2008), Bitzer (2004) and Hasnas, Lambertini and Palestini (2014).

papers, in terms of the decision a firm makes. In our model, firms decide whether to join the available open source community or not, whereas in these studies mentioned the firm either decides whether to make its *own* source open or not, or support open source development or not. Also, unlike these studies that focus on duopolistic structures, we allow for many firms and study the evolution of open source, through open source use decisions, when the set of firms is not small.

Among other R&D related studies, Reisinger, Ressner, Schmidtke and Thomes (2014) consider a model where firms produce a private good and invest in the quality of a public good, like an open source project.⁹ Erkal and Minehart (2014) study the dynamic games of R&D, where they explore the effect of knowledge sharing and incentives to license the intermediate steps as they approach the product market competition.¹⁰ Our paper also differs from these papers, in either modeling aspects or in the main question posed or the firms being allowed to make profit or not. We believe that it is important to characterize the behavior of a firm when there is open source subject to GPL, both in terms of its open source use decision, its investment in innovation and its profit. Our model is both dynamic and tractable, and it is capable of capturing the essence of the GPL licensing.

Section 2 depicts the model. Section 3 solves the model and provides the main results. In Section 4, we discuss some relevant points and extensions. Section 5 concludes.

2 The Model

There are $M \geq 2$ firms interacting over $T \geq 3$ many periods. In each period t , each firm i produces a good at a firm-period specific unit cost c_i^t , which is stochastically determined by firm i 's investment in cost-reducing innovation. There is also a public production tech-

⁹They find that, under super additive investment cost function, if there is an additional firm or there is a government contribution to the public good, then all firms increase their contributions, that is, they get a crowd-in effect.

¹⁰See also Grossman and Shapiro (1987), Aoki (1991) and Fershtman and Malkovich (2010).

nology, called *open source*, which can produce the good in period t at a unit cost $c_{os}^t > 0$, which is also stochastically determined by the open source using firms' investments in cost-reducing innovation.

Chain of events *within* a period: In each period, there are three stages:

- (1) each firm decides whether to adopt the open source or not,
- (2) each firm invests in cost-reducing innovation,
- (3) firms compete in quantities in a Cournot fashion.

To capture the effect of open source under GPL, we make the following assumptions.

Assumption 1 *Each firm is free to use the open source at no direct cost.*

Assumption 2 *Any innovation made by a firm which uses the open source in period t , will be open source from period $t + 1$ on.*

Production cost: To be more precise, let $k_i^t \in \mathbb{Z}_+$ denote the production technology level for firm i at the beginning of period t . The unit cost of firm i is given by the function $c_i^t = c(k_i^t) = \frac{1}{k_i^t + 1}$.¹¹ Thus, the unit cost strictly decreases as the production technology level increases and in the limit as k_i goes to infinity, the marginal cost goes to zero. Let $k_{os}^t \in \mathbb{Z}_+$ denote the production technology level of the open source at the beginning of period t . That is, before period t starts, the public production technology level is able to produce the good at a unit cost $c_{os}^t = \frac{1}{k_{os}^t + 1}$. Likewise, k_{os}^1 is the initial technology level

¹¹An alternative marginal cost function, for instance, is the one used in Modica (2012): when there is innovation the new unit cost is $c_{new} = (1 - b)c_0$, where $b \in (0, 1)$ and c_0 being the initial unit cost. Also, Aghion, Harris, Howitt and Vickers (2001), looking at the effect of production market competition and imitation on growth, assume that a firm's unit cost depends on its technology level and when a firm advances its technology level by one step, its unit cost decreases by some factor. However, since our model is dynamic, we want to have a tractable cost reduction process as a function of the technology level. With $c_{new} = (1 - b)c$ type of reduction process, we would need to have $c(k) = (1 - b)c(k - 1) = (1 - b)^2 c(k - 2) = (1 - b)^k c_0$, which is not analytically tractable in our model.

of the open source at the beginning of period 1.

Open source use decisions: At the first stage of each period t , each firm i who has not used the open source before decides whether to use the open source or not. Let $d_i^t \in \{0, 1\}$ denote this open source use decision of firm i at the first stage of period t , where 1 stands for *use* decision and 0 stands for *not use* decision. We call a firm with $d_i^t = 0$ a *non-user* firm and a firm with $d_i^t = 1$ a *user* firm. When a firm is indifferent between *using* and *not using* the open source, we assume that it chooses to use it. If a firm j has already used the open source at some period t' , then $d_j^t = 1$ for each $t \geq t'$.¹² Also, a firm can freely use the open source at no direct cost (Assumption 1 above). Let $\kappa_i^t(d_i, k_i)$ denote the technology level of firm i after its open source use decision d_i^t before which it had a technology level k_i . That is,

$$\kappa_i^t(d_i, k_i) = d_i^t \max(k_i^t, k_{os}^t) + (1 - d_i^t) k_i^t.$$

Note that $\kappa_{os}^t = k_{os}^t$ for each t , since the open source has no *use-not use* decision.

Investment in innovation: At the second stage of each period t , each firm i decides how much to invest in innovation. We model investment in innovation through probability of success: firm i with a technology level k_i^t and open source use decision d_i^t , picks a probability of success, $p(d_i^t, k_i^t)$,¹³ at a cost $C(p(d_i^t, k_i^t))$ with $C' > 0$ and $C'' > 0$. With probability $p(d_i^t, k_i^t)$ there is success and the firm advances one level in the technology ladder, that is, achieves a new level, $\kappa_i^t + 1$, and reduces its unit cost from $\frac{1}{\kappa_i^t + 1}$ to $\frac{1}{\kappa_i^t + 2}$.

¹²Here we assume that a firm who has already joined the open source community cannot leave it. A more general way to model it would be to allow the firms to leave the open source community whenever they want, and show that they will not leave it in the equilibrium. We discuss this in the Discussion and Extensions section.

¹³We write $p(d_i^t, k_i^t)$ instead of $p(\kappa_i^t)$, since two firms with the same κ_i^t might choose different probabilities if they have different d_i^t .

With probability $1 - p(d_i^t, k_i^t)$ the firm fails to advance one level and stays at the current technology level κ_i . We denote the realization of the new technology level with K_i^t .

If a firm is using the open source at period t , then its technology level depends on the open source's current technology level and its own success/failure outcome. More precisely,

$$K_i^t = \begin{cases} \kappa_i^t + 1 & \text{with probability } p(d_i^t, k_i^t) \\ \kappa_i^t & \text{with probability } 1 - p(d_i^t, k_i^t) \end{cases}$$

If $d_i^t = 1$, then

$$K_i^t = \begin{cases} \max(k_i^t, k_{os}^t) + 1 & \text{if success} \\ \max(k_i^t, k_{os}^t) & \text{if fail} \end{cases}$$

The technology level of the open source depends on its previous period technology level and the previous period's success/failure outcome of the firms that were using the open source. That is, the successful innovation by a user firm is reflected on the open source with exactly one period lag (Assumption 2 above). More precisely,

$$K_{os}^t = k_{os}^t = \begin{cases} k_{os}^{t-1} + 1 & \text{with probability } 1 - [1 - p(1, k_{os}^{t-1})]^{n_1(k_{os}^{t-1}, t)} \\ k_{os}^{t-1} & \text{with probability } [1 - p(1, k_{os}^{t-1})]^{n_1(k_{os}^{t-1}, t)} \end{cases}$$

where $n_1(k_{os}^{t-1}, t - 1)$ is the number of firms with $d_i^{t-1} = 1$ in period $t - 1$.

Cournot competition: At the third stage of each period t , firms engage in quantity competition a la Cournot. Each firm i simultaneously decides how much to produce, $q_{K_i^t}$, when it has a technology level K_i^t at the beginning of the third stage. A firm with K_i^t has an expected inverse demand given by $\mathbf{P}_{K_i^t} \equiv P(\mathbf{Q}_{K_i^t}) = A - \mathbf{Q}_{K_i^t}$, where $A > 0$ is sufficiently large, P is the market price, and $\mathbf{Q}_{K_i^t}$ is the total quantity demanded that a firm with K_i^t expects. This total quantity can be decomposed into two parts, its own quantity,

which is known, and expected total quantity of all other firms, that is, $\mathbf{Q}_{K_i^t} = q_{K_i^t} + \mathbf{Q}_{-i}^t$. At the end of this stage, each firm realizes its profit level $\pi_{K_i^t}^t$ in the Cournot competition.

Overall payoff of a firm: Firm i 's overall payoff is its discounted sum of within period Cournot profits and cost of investment. That is,

$$\Pi_i = \sum_{t=1}^T \delta^{t-1} [\pi_{K_i^t}^t - C(p(d_i^t, k_i^t))] \quad (1)$$

where δ is the discount factor of firm i .

Distribution of firms: At the beginning of the first stage of period $t = 1$, there are $n(k, 1) \neq 0$ firms with unit cost $c(k)$ where $\sum_k n(k, 1) = M$ and $k \in \{0, 1, 2\}$. We assume that the initial technology level of the open source is $k_{os}^1 = 1$. Thus, there are firms which are at a lower technology level than the open source, firms which are at a higher technology level than the open source, and firms which are at the same technology level as the open source. The number of firms that have the technology level $k_{os}^1 = 1$ at period $t = 1$, $n(1, 1)$, is composed of user and non-user firms: $n(1, 1) = n_0(1, 1) + n_1(1, 1)$, where $n_0(1, 1)$ and $n_1(1, 1)$ denote the number of non-user and user firms, respectively. We assume that each of $n(0, 1)$, $n_0(1, 1)$, $n_1(1, 1)$ and $n(2, 1)$ are publicly observed at the beginning of the first stage.

At the beginning of the first stage in period $t > 1$, there are $n(k, t)$ firms with unit cost $c(k)$ where $k \in \{0, 1, \dots, t + 2\}$. For each k , $n(k, t) = n_0(k, t) + n_1(k, t)$, where $n_0(k, t)$ and $n_1(k, t)$ denote the number of non-user and user firms, respectively.¹⁴ We assume that for each k , $n_0(k, t)$ and $n_1(k, t)$, thus $n(k, t)$, are all publicly observed at the beginning of each period t . Note that it is possible that $n(k, t) = 0$ for some k , for instance, when

¹⁴Note that if $k < k_{os}$ or $k > k_{os} + 1$, then $n_1(k, t) = 0$. When $k = k_{os} + 1$, there may be user firms who have succeeded in innovation in the previous period.

$k_{os}^t > k$ and there is no non-user firm.

At the beginning of the second stage of period $t \geq 1$, after the open source use decisions have been made, the number of firms with technology level κ is denoted by $\eta(\kappa, t)$. At this stage, let $\eta_1(\kappa, t)$ and $\eta_0(\kappa, t)$ denote the number of user firms and non-user firms respectively, that is, $\eta(\kappa, t) = \eta_0(\kappa, t) + \eta_1(\kappa, t)$. Note that whenever $\kappa \neq \kappa_{os}$, $\eta_1(\kappa, t) = 0$.

At the beginning of the third stage of period $t \geq 1$, after the success/failure outcomes are realized, the number of firms that have technology level K is denoted by $N(K, t)$. Similarly, $N(K, t) = N_0(K, t) + N_1(K, t)$, where $N_0(K, t)$ and $N_1(K, t)$ denote the number of non-user and the number of user firms, respectively.

To sum up, in a given period t , the technology level of a firm is k_i^t at the beginning of the first stage, κ_i^t at the beginning of the second stage, and K_i^t at the beginning of the third stage, where $i = os$ denotes these technology levels for the open source. And, at any given period t , $n(k, t)$ is the number of firms with technology level k at the beginning of the first stage, $\eta(\kappa, t)$ is the number of firms with the technology level κ at the beginning of the second stage, and $N(K, t)$ is the number of firms with the technology level K at the beginning of the third stage. Note that, $\eta(\cdot, t) \geq n(\cdot, t)$, $n(\cdot, t+1) = N(\cdot, t)$ and $n_d(\cdot, t+1) = N_d(\cdot, t)$ for each $d \in \{0, 1\}$ and $t \geq 1$.

3 Equilibrium Analysis

To solve this model, we study the Subgame Perfect Nash equilibrium of the 3 stage - T period game. We focus on symmetric equilibria, that is, we assume that in the third stage all firms with the same technology level pick the same quantity, in the second stage all firms with the same use decision and technology level pick the same success probability, and finally in the first stage all non-user firms with the same technology level make the same open source use decision.

3.1 Last Period: $t=T$

Before we start the equilibrium analysis with the last stage of the last period, we provide a lemma first, which deals with the first stage of the last period.

Lemma 1 $d_i^T = 1$ is a best response for any history of the game, for all i .

Proof. The proof is straightforward. Any firm with $k_i^T < k_{os}^T$ in the beginning of period T will be strictly better off using the open source since it will strictly lower its unit cost. Any firm with $k_i^T = k_{os}^T$ will be indifferent between using the open source and not using it. Any firm with $k_i^T > k_{os}^T$ has no benefit from using the open source. Also, since there is no future periods in the last period, there is no negative future effect of using the open source as well.¹⁵ Thus, such firms will also be indifferent between using and not using the open source. Thus, all firms at T will be weakly better off using the open source. ■

Now we turn to the last stage of the last period. We denote the equilibrium number of firms of a certain technology level and thus a firm's expectation of the equilibrium number of firms of that technology level with the boldface counterparts. For instance, $\mathbf{N}(\mathbf{K}, \mathbf{t})$ denotes the equilibrium number of firms with technology level K at the beginning of the third stage of period t and also the expected number of such firms.

3.1.1 Third Stage: Cournot Competition

Each firm i observes own unit cost K_i^T but does not observe the innovation outcome of the other firms. Thus, each firm has an expectation of the technology distribution in the market, $\{\mathbf{N}(\mathbf{K}, \mathbf{T})\}_K$. The expected inverse demand with K_i is $\mathbf{P}_{K_i^T} \equiv P(\mathbf{Q}_{K_i^T}) =$

¹⁵Even if such firms choose not to use the open source the distribution of firms according to the technology levels will be the same as in the case where such firms choose to use the open source. Thus, it does not affect the rest of the analysis.

$A - \mathbf{Q}_{K_i^T}$. The expected total quantity can be decomposed; $\mathbf{Q}_{K_i^T} = q_{K_i^T} + \mathbf{Q}_{-i}^T$ where $\mathbf{Q}_{-i}^T = \sum_{K \neq K_i^T} \mathbf{N}(\mathbf{K}, \mathbf{T}) q_{K^T} + (\mathbf{N}(\mathbf{K}_i^T, \mathbf{T}) - 1) q_{K_i^T}$.

By Lemma 1, there is no firm with $K_i^T < K_{os}^T$ after innovation realizations in the last period. Thus, in the equilibrium we have $K_i^T \in \{K_{os}^T, K_{os}^T + 1, \dots, T + 2\}$ for each i . When a firm i with K_i^T chooses its quantity $q_{K_i^T}^T$, it solves the following problem

$$\max_q E[\pi_{K_i^T}^T] = (\mathbf{P}_{K_i^T} - c_{K_i^T}^T)q$$

Then, the equilibrium quantities are as follows;¹⁶

$$q_{K_{os}^T} = \frac{A}{M+1} - \frac{1}{(M+1)(K_{os}^T+1)} - \frac{1}{(M+1)} \sum_{t=1}^{T+2-K_{os}^T} \mathbf{N}(\mathbf{K}_{os}^T + \mathbf{t}, \mathbf{T}) \left(\frac{t}{(K_{os}^T+1)(K_{os}^T+t+1)} \right)$$

and

$$q_{K_i^T} = q_{K_{os}^T} + \frac{1}{K_{os}^T+1} - \frac{1}{K_i^T+1} \quad (2)$$

for each $K_i^T > K_{os}^T$. Note that if $K > K'$, then $q_K > q_{K'}$, since $\frac{1}{K'+1} - \frac{1}{K+1} > 0$. The expected Cournot profit levels at this stage are $E[\pi_{K_i^T}^T] = (q_{K_i^T}^T)^2$ for each $K_i^T \geq K_{os}^T$.¹⁷ Thus, the more advanced a firm is at the technology ladder, the more it produces and the higher expected Cournot profit it gets. This is of no surprise because the more advanced firms have lower unit costs, thus they have cost advantage in the Cournot competition. Thus, they produce more in the equilibrium and end up with higher profits in the Cournot competition.

¹⁶For details, please see the Appendix.

¹⁷Also see the Appendix.

3.1.2 Second Stage: Investment in Innovation

In the second stage, firms decide their investment levels by picking probability of success to advance a level in the technology level, that is, to decrease the unit cost. Each firm knows own unit cost at the beginning of this stage. Firms, after observing $n(k, t)$ for each k , do not observe the open source use decisions of other firms, d_j^t . They, however, know the expected number of firms for each k , $\eta(\mathbf{k}, \mathbf{t})$. Like the third stage, in this innovation stage, the technology level a firm has is at least $\kappa_{os}^T = k_{os}^T$ by Lemma 1. Thus, $\kappa_i^T \in \{k_{os}^T, k_{os}^T + 1, \dots, T + 1\}$. Firm i with a technology level k_i^T and open source use decision d_i^T picks a probability of success of innovation, $p(d_i^T, k_i^T) \in (0, 1)$, by maximizing its expected profit. That is,

$$\max_p pE[\pi_{\kappa_{i+1}}^T] + (1 - p)E[\pi_{\kappa_i}^T] - C(p)$$

Note that by Lemma 1, $d_i^T = 1$ for each i . Thus,

$$\kappa_i^T(d_i^T, k_i^T) = d_i^T \max(k_i^T, k_{os}^T) + (1 - d_i^T)k_i^T = \max(k_i^T, k_{os}^T)$$

is firm i 's (potentially) new technology level.

The equilibrium probability of firm i with k_i^T and $d_i^T = 1$ is then given by

$$\begin{aligned} C'(p(d_i^T, k_i^T)) &= E[\pi_{\kappa_{i+1}}^T] - E[\pi_{\kappa_i}^T] \\ &= q_{\kappa_i^T+1}^2 - q_{\kappa_i^T}^2 \\ &= (q_{\kappa_i^T+1} + q_{\kappa_i^T})(q_{\kappa_i^T+1} - q_{\kappa_i^T}) \\ &= \left(2q_{K_{os}^T} + \frac{2}{K_{os}^T + 1} - \frac{1}{\kappa_i^T + 2} - \frac{1}{\kappa_i^T + 1}\right) \left(-\frac{1}{\kappa_i^T + 2} + \frac{1}{\kappa_i^T + 1}\right) \end{aligned}$$

When $d_i^T = 1$ by Lemma 1, we have $\kappa_i^T = k_{os}^T$. Also, $k_{os}^T = K_{os}^T$ since the innovation

success by a user firm is not reflected until next period (in this case, it is never reflected because T is the last period). Then we get,

$$\begin{aligned} C'(p(1, k_i^T)) &= (2q_{K_{os}^T} + \frac{2}{K_{os}^T + 1} - \frac{1}{K_{os}^T + 2} - \frac{1}{K_{os}^T + 1})(-\frac{1}{K_{os}^T + 2} + \frac{1}{K_{os}^T + 1}) \\ &= [2q_{K_{os}^T} + \frac{1}{(K_{os}^T + 1)(K_{os}^T + 2)}] \frac{1}{(K_{os}^T + 1)(K_{os}^T + 2)} \end{aligned}$$

3.1.3 First Stage: Using Open Source

The first stage in period T is already discussed in the beginning of this section. And the result is summarized in Lemma 1: each firm (weakly) prefers to use the open source in the last period.

3.2 Next to Last Period: $t=T-1$

3.2.1 Third Stage: Cournot Competition

Each firm i observes own unit cost K_i^{T-1} , but does not observe the innovation outcome of the other firms and has an expectation of the technology distribution in the market, $\{\mathbf{N}(\mathbf{K}, \mathbf{T} - 1)\}_K$. Again, the expected inverse demand with K_i is $\mathbf{P}_{K_i^{T-1}} \equiv P(\mathbf{Q}_{K_i^{T-1}}) = A - \mathbf{Q}_{K_i^{T-1}}$, where $\mathbf{Q}_{K_i^{T-1}} = q_{K_i^{T-1}} + \mathbf{Q}_{-i}^{T-1}$ where $\mathbf{Q}_{-i}^{T-1} = \sum_{K \neq K_i^{T-1}} \mathbf{N}(\mathbf{K}, \mathbf{T} - 1)q_{K^{T-1}} + (\mathbf{N}(\mathbf{K}_i^{T-1}, \mathbf{T} - 1) - 1)q_{K_i^{T-1}}$.

A firm i with K_i^{T-1} chooses its quantity $q_{K_i^{T-1}}$ to solve the following problem

$$\max_q E[\pi_{K_i^{T-1}}] = (\mathbf{P}_{K_i^{T-1}} - c_{K_i^{T-1}})q$$

A notable difference from the last period is that $K_i^{T-1} \in \{0, 1, \dots, T + 1\}$ is the set of possible technology levels. The equilibrium quantities are as follows:¹⁸

¹⁸The analysis is the same as in period T for the details of which, please see Appendix.

$$q_{0^{T-1}} = \frac{A-1}{M+1} - \frac{1}{(M+1)} \sum_{t=1}^{T+1} \mathbf{N}(\mathbf{t}, \mathbf{T} - \mathbf{1}) \frac{t}{t+1}$$

and

$$q_{K_i^{T-1}} = q_{0^{T-1}} + \frac{K_i^{T-1}}{K_i^{T-1} + 1} \quad (3)$$

for each $K_i^{T-1} > 0$.

The expected Cournot profit levels at this stage are similar to those in the last period: $E[\pi_{K_i^{T-1}}] = (q_{K_i^{T-1}})^2$ for each $K_i^{T-1} \geq 0$.

3.2.2 Second Stage: Investment in Innovation

For firm i with a technology level k_i^{T-1} and open source use decision d_i^{T-1} ,

$$\kappa_i^{T-1}(d_i^{T-1}, k_i^{T-1}) = d_i^{T-1} \max(k_i^{T-1}, k_{os}^{T-1}) + (1 - d_i^{T-1})k_i^{T-1} \quad (4)$$

is its (potentially) new technology level. Such a firm picks a probability of success of innovation, $p(d_i^{T-1}, k_i^{T-1}) \in (0, 1)$, by maximizing its expected profit. That is,

$$\max_p p [E[\pi_{\kappa_i^{T-1}}] + \delta W_S(d_i^{T-1}, \kappa_i^{T-1} + 1)] + (1 - p) [E[\pi_{\kappa_i^{T-1}}] + \delta W_F(d_i^{T-1}, \kappa_i^{T-1})] - C(p)$$

where $W_S(d_i^{T-1}, \kappa_i^{T-1} + 1)$ is the expected continuation payoff from period T on, when at $T - 1$ the open source use decision is d_i^{T-1} and the technology level at the end of period $T - 1$ is $\kappa_i^{T-1} + 1$, with a success in innovation in the investment stage. Similarly, $W_F(d_i^{T-1}, \kappa_i^{T-1})$ is the expected continuation payoff from period T on, when at $T - 1$ the open source use decision is d_i^{T-1} and the technology level at the end of period $T - 1$ is κ_i^{T-1} , with a failure in innovation in the investment stage. Note that in this investment stage, possible technology levels are $\kappa_i^{T-1} \in \{0, 1, \dots, T\}$.

Similar to period T , the equilibrium investment probability of firm i with k_i^{T-1} and

d_i^{T-1} is then given by

$$\begin{aligned} C'(p(d_i^{T-1}, k_i^{T-1})) &= E[\pi_{\kappa_i+1}^{T-1}] - E[\pi_{\kappa_i}^{T-1}] + \delta [W_S(d_i^{T-1}, \kappa_i^{T-1} + 1) - W_F(d_i^{T-1}, \kappa_i^{T-1})] \\ &= q_{\kappa_i^{T-1}+1}^2 - q_{\kappa_i^{T-1}}^2 + \delta [W_S(d_i^{T-1}, \kappa_i^{T-1} + 1) - W_F(d_i^{T-1}, \kappa_i^{T-1})] \end{aligned}$$

3.2.3 First Stage: Using Open Source

In this stage, each firm decides whether to use the open source or not. The decision depends on the current own technology level k_i^{T-1} , the current technology level of the open source, k_{os}^{T-1} , and the distribution of non-user and user firms for each technology level k , $n_0(k, T-1)$ and $n_1(k, T-1)$. The main trade-off for a non-user firm with $k_i^{T-1} < k_{os}^{T-1}$ is as follows. Starting to use the open source in this period will be beneficial for the current period through lower unit cost thus higher Cournot profit. Also, in case of a failure, success of a user firm will carry the technology level of one step higher and the firm will be able to use it from next period on. However, a potential innovation by such a firm will decrease the unit cost of other firms in the next period, because of the structure imposed by the GPL, and that will provide the other firms with the same cost advantage in the Cournot competition in the current period.

Let $V(d_i^{T-1}, k_i^{T-1})$ denote the expected payoff of firm i from period $T-1$ on. That is,

$$\begin{aligned} V(d_i^{T-1}, k_i^{T-1}) &= p(d_i^{T-1}, k_i^{T-1}) [E[\pi_{\kappa_i+1}^{T-1}] + \delta W_S(d_i^{T-1}, \kappa_i^{T-1} + 1)] \\ &+ (1 - p(d_i^{T-1}, k_i^{T-1})) [E[\pi_{\kappa_i}^{T-1}] + \delta W_F(d_i^{T-1}, \kappa_i^{T-1})] \\ &- C(p(d_i^{T-1}, k_i^{T-1})) \end{aligned}$$

We show that the optimal open source use decision for a firm with $k_i^{T-1} < k_{os}^{T-1}$ is to use the open source, and the optimal open source use decision for a firm with $k_i^{T-1} \geq k_{os}^{T-1}$

is not to use the open source. These are summarized, respectively, in the two propositions below.

Proposition 1 *If $k_i^{T-1} \geq k_{os}^{T-1}$, then $d_i^{T-1} = 0$.*

Proof. See the Appendix. ■

The intuition for this result is that whenever a firm starts the game with the same unit cost as the open source has, there is no direct benefit from using the open source. However, using the open source makes the firm obliged to share its potential first period innovation, with other firms, in the second period, who choose to use the open source in the second period. This removes any potential cost advantage the firm could have in the second period quantity setting game. Hence any such firm will avoid using the open source.

Our second result says that any firm that produces the good at a higher unit cost than the open source will choose to use the open source even though GPL makes the firm obliged to share its potential innovation in the first period with the firms in the second period.

Proposition 2 *If $k_i^{T-1} = k_{os}^{T-1} - 1$, then $d_i^{T-1} = 1$.*

Proof. See the Appendix. ■

The intuition for this result is as follows. For a firm with a lower technology level (that is, a higher unit cost) than the open source, using the open source directly improves the firm's production technology hence its expected profit in period $T - 1$. However, using the open source now and succeeding in cost reduction in the current period will provide the other users who have not succeeded with the cost reduction in the next period. Not using the open source now, and succeeding both now and in the next period may result in a cost

advantage in the next period if the open source fails both now and later. Thus, there is a tradeoff. The direct effect dominates the negative effect of using the open source, since the negative effect is of second degree. Therefore, the incentive to use the open source dominates the incentive not to use it.

Corollary 1 *If $k_i^{T-1} < k_{os}^{T-1}$, then $d_i^{T-1} = 1$.*

This is a direct implication of Proposition 2. The reason is as follows. Compared to the case $k_i^{T-1} = k_{os}^{T-1} - 1$, the positive/immediate effect of using the open source is larger than the one for case $k_i^{T-1} < k_{os}^{T-1} - 1$. However, the negative/future effect is the same in both cases. Thus, any firm who has a technology level less than the open source technology level, will prefer to start using the open source.

3.3 Any period $t < T$

Now we argue that the Proposition 1 is valid for any period $t < T$.

Proposition 3 *If $k_i^t \geq k_{os}^t$, then $d_i^t = 0$.*

Proof. The proof is similar to the proof of Proposition 1. ■

We also argue that both the Proposition 2 and its corollary are valid for any period $t < T$. Thus we prove the following proposition.

Proposition 4 *If $k_i^t < k_{os}^t$, then $d_i^t = 1$.*

Proof. The proof is similar to the proof of Proposition 2. Since we know that in period $T - 1$ each firm with a lower technology level than the open source's level, will use the open source. Thus, at period $T - 1$ each firm will have at least open source technology level. Thus, at $T - 2$, the reasoning to use the open source will be similar to the one in the proof of Proposition 2. ■

An immediate implication of the proposition above is

Corollary 2 *Each period $t \leq T$, the distribution of firms after the open source use decisions are made is such that $\eta(k, t) = 0$ for all $k < k_{os}^t$.*

Thus, putting all these together we get the following Proposition.

Proposition 5 *At the beginning of the first stage of any period $1 \leq t < T$, $k_i^t \geq k_{os}^t - 1$. If $k_i^t \geq k_{os}^t$, then $d_i^t = 0$. If $k_i^t = k_{os}^t - 1$, then $d_i^t = 1$.*

Thus, each firm stays out of the open source as long as it's not behind the open source. A firm which has the same technology level as the open source may fail while the open source may succeed, thus next period the firm is one step behind the open source. At that point the firm starts using the open source.

In terms of the evolution of the open source community, in the light of the above results, as long as the open source users are successful in innovation, the open source community will grow and sweep the non-users, and the set of non-user firms will shrink. As long as, the open source is not always successful in innovation, and the non-user firms that are ahead of the open source technology level succeed, there will be a set of proprietary firms with a higher technology level.

4 Welfare

One of the most important questions is whether the open innovation under GPL improves social welfare of the economy and, if it does, by how much. Based on firm's optimal use decision, our model provides a way to measure the welfare gain and loss from open source. The welfare gain comes from stronger competition in the production stage than without open source case. In the innovation race, open source helps firms that are *behind* to keep chasing firms that are *ahead*. On the other hand, it increases the likelihood that

advanced firms are caught relative to the situation without open source. Then, there will be larger consumer surplus under the open source than without it. However, firms have investment costs as well as production costs. A stronger competition under the open source may lead both advanced firms and lagged firms invest more than the socially optimal level. We can measure how much social welfare gain comes from the open source as $SW = SW(k_{os} = 1) - SW(k_{os} = 0)$ where $SW(k_{os}) = CS(k_{os}) + PS(k_{os})$, CS is consumer surplus, and PS is producer surplus;

$$\begin{aligned}
CS(k_{os}) &= \frac{1}{2}(A - P)Q + \frac{1}{2}(A - P')Q' \\
&= \frac{1}{2}(A - (A - Q))Q + \frac{1}{2}(A - (A - Q'))Q' \\
&= \frac{1}{2}(Q^2 + Q'^2) \\
&= \frac{1}{2}\left[\left(\sum_{k=0}^2 N_k q_k\right)^2 + \left(\sum_{k=1}^3 N'_k q'_k\right)^2\right]
\end{aligned}$$

and

$$\begin{aligned}
PS(k_{os}) &= \sum_{k=0}^2 N_k [\pi_k - C(p_k)] + \sum_{k=1}^3 N'_k [\pi'_k - C(p'_k)] \\
&= \sum_{k=0}^2 N_k \left[(q_k)^2 - \delta \left(q_k + \frac{\delta}{4} \right) \right] + \sum_{k=1}^3 N'_k \left[(q'_k)^2 - \delta \left(q'_k + \frac{\delta}{4} \right) \right].
\end{aligned}$$

5 Discussion and Extensions

In this section we discuss several relevant points and extensions.

The initial unit cost distribution we have assumed is a specific one; k_i is equal to either 0,1 or 2, where the open source is at a level $k_{os} = 1$, to allow to have firms both at a lower

and a higher technology levels relative to the open source. Instead, we can assume a more general distribution. The initial technology levels of the firms can be $k \in \{0, 1, \dots, K\}$ and $k_{os} \in \{0, 1, \dots, K - 1\}$, where $k_{os} \neq K$ ensures that there are firms with an initial technology level higher than the open source's level. Under this generalization, we believe that the idea behind our results will still be valid, that is, the firms that have a higher unit cost (lower technology level) than the source will use the open source, the firms that have the same or lower unit cost will choose not to use the source.

Also, we assumed that the firms who start using the open source they do not leave the open source. However, one could relax this assumption and let the users of open source leave it whenever they want. Under this specification, if, in the symmetric equilibrium, at any period if the user firms choose to stay in the open source, then the outcome is equivalent to what we have provided in our equilibrium analysis above. However, if the user firms decide to leave the open source the following period, then the number of user firms will be less relative to the specification in our model. Thus, the open source will evolve relatively slower, in expectations. But, we believe that the use/not use decisions of the behind and ahead firms will not be affected.

We have used a specific cost reduction function, $c(k) = \frac{1}{k+1}$. It would be a natural extension if we use a more general cost reduction function that decreases in the technology level and approaches to zero as the technology level goes to infinity. However, we believe that our results will go through with more general cost reduction functions.

6 Conclusion

In this paper, we analyzed a simple dynamic model of innovation in cost reduction with an open source production technology present for the firms to freely use. We assumed, in the spirit of the GPL, that whenever a user succeeds in cost reduction innovation, it

has to share this new technology with other users, in the subsequent periods. Because of this aspect of the GPL, we used a dynamic model with T many periods, where in each period the firms decide whether to use the open source or not, invest in innovation, and engage in quantity competition. We characterized the optimal open source use decision of a firm as a function of its technology level relative to the open source. A firm that has the same (or higher) technology as the open source finds it optimal not to use the open source. A firm that has a lower production technology level finds it optimal to use the source. These results show what principal effects of open source license on incentives for innovation and usage of the open source are. We believe that our model can be used as a tractable tool for further studies in open innovation.

REFERENCES

1. Admati, A. R. and Perry, M. (1991), “Joint Projects without Commitment”, *The Review of Economic Studies*, 58, 259-276.
2. Aghion, P., Harris, C., Howitt, P. and Vickers, J. (2001), “Competition, Imitation and Growth with Step-by-Step Innovation”, *Review of Economic Studies*, 68, 467-492.
3. Aoki, R. (1991), “R&D Competition for Product Innovation: An Endless Race”, *The American Economic Review*, 81, 252-256.
4. Atal, V. and Shankar, K. (2014), “Open Source Software: Competition with A Public Good”, *Atlantic Economic Journal*, 42(3), 333-345.
5. Athey, S. and Ellison, G. (2014), “Dynamics of Open Source Movements”, *Journal of Economics & Management Strategy*, 23(2), 294-316.

6. Bitzer, J. (2004), “Commercial versus open source software: the role of product heterogeneity in competition”, *Economic Systems*, 28, 369-381.
7. Bitzer, J., Schrettl, W. and Schröder, P. J. H. (2007), “Intrinsic motivation in open source software development”, *Journal of Comparative Economics*, 35, 160-169.
8. Casadesus-Masanell, R. and Ghemawat, P. (2006), “Dynamic Mixed Duopoly: A Model Motivated by Linux vs. Windows”, *Management Science*, 52(7), 1072-1084.
9. Casadesus-Masanell, R. and Llanes, G. (2011), “Mixed Source”, *Management Science*, 57(7), 1212-1230.
10. Caulkins, J. P., Feichtinger, G., Grass, D., Hartl, R. F., Kort, P. M., and Seidl, A. (2013), “When to make proprietary software open source”, *Journal of Economic Dynamics & Control*, 37, 1182-1194.
11. Economides, N. and Katsamakas, E. (2006), “Two-Sided Competition of Proprietary vs. Open Source Technology Platforms and the Implications for the Software Industry”, *Management Science*, 52(7), 1057-1071.
12. Erkal, N. and Minehart, D. (2014), “Optimal Technology Sharing Strategies in Dynamic Games of R&D”, *Journal of Economics & Management Strategy*, 23(1), 149-177.
13. Fershtmana, C. and Markovich, S. (2010), “Patents, imitation and licensing in an asymmetric dynamic R&D race”, *International Journal of Industrial Organization*, 28(2), 113-126.
14. Grossman, G. M. and Shapiro, C. (1987), “Dynamic R&D Competition”, *The Economic Journal*, 97, 372-387.

15. Haruvy, E., Sethi, S. P. and Zhou, J. (2008), “Open Source Development with a Commercial Complementary Product or Service”, *Production and Operations Management*, 17(1), 29-43
16. Hasnas, I., Lambertini, L. and Palestini, A. (2014), “Open Innovation in a dynamic Cournot duopoly”, *Economic Modelling*, 36, 79-87.
17. Jaisingh, J, See-To, E. W. K. and Tam K. Y. (2008), “The Impact of Open Source Software on the Strategic Choices of Firms Developing Proprietary Software”, *Journal of Management Information Systems*, 25(3), 241-276.
18. Johnson, J. P. (2002), “Open Source Software: Private Provision of a Public Good”, *Journal of Economics & Management Strategy*, 11(4), 637-662.
19. Kort, P. M. and Zaccour, G. (2011), “When Should a Firm Open its Source Code: A Strategic Analysis”, *Production and Operations Management*, 20(6), 877-888.
20. Lerner, J., Pathak, P. A. and Tirole, J. (2006), “The Dynamics of Open-Source Contributors”, *The American Economic Review*, 96(2), 114-118.
21. Lerner, J. and Tirole, J. (2005), “The Economics of Technology Sharing: Open Source and Beyond”, *The Journal of Economic Perspectives*, 19(2), 99-120.
22. Lerner, J. and Tirole, J. (2002), “Some Simple Economics of Open Source”, *Journal of Industrial Economics*, 50(2), 197-234.
23. Li, C. and Ji, C. (2010), “Innovation, licensing, and price vs. quantity competition”, *Economic Modelling*, 27, 746-754.
24. Llanes, G. and de Elejalde, R. (2013), “Industry equilibrium with open-source and proprietary firms”, *International Journal of Industrial Organization*, 31, 36-49.

25. Marx, L. E. and Matthews, S. A. (2000), “Dynamic Voluntary Contribution to a Public Project”, *The Review of Economic Studies*, 67, 327-358.
26. Modica, S. (2012), “Open Source without Free-Riding”, *Economia politica*, 2, 247-260.
27. Mustonen, M. (2005), “When Does a Firm Support Substitute Open Source Programming?”, *Journal of Economics & Management Strategy*, 14(1), 121-139.
28. Reisinger, M., Ressler, L., Schmidtke, R. and Thomes, T. P. (2014), “Crowding-in of complementary contributions to public goods: Firm investment into open source software”, *Journal of Economic Behavior & Organization*, 106, 78-94.
29. Yildirim, H. (2006), “Getting the ball rolling: Voluntary contributions to a large-scale public project”, *Journal of Public Economic Theory*, 8(4), 503-528.

7 Appendix

7.1 Cournot Competition at $t=T$

Recall that the expected inverse demand with K_i is $\mathbf{P}_{K_i^T} \equiv P(\mathbf{Q}_{K_i^T}) = A - \mathbf{Q}_{K_i^T}$. The expected total quantity can be decomposed; $\mathbf{Q}_{K_i^T} = q_{K_i^T} + \mathbf{Q}_{-i}^T$ where $\mathbf{Q}_{-i}^T = \sum_{K \neq K_i^T} \mathbf{N}(\mathbf{K}, \mathbf{T})q_{K^T} + (\mathbf{N}(\mathbf{K}_i^T, \mathbf{T}) - 1)q_{K_i^T}$. Under Cournot competition, a firm i with K_i^T solves the following problem

$$\max_q E[\pi_{K_i^T}^T] = (\mathbf{P}_{K_i^T} - c_{K_i^T}^T)q$$

The first order condition is

$$\begin{aligned}
\frac{\partial E[\pi_{K_i^T}]}{\partial q} &= P(\mathbf{Q}_{K_i^T}) - c_{K_i}^T + \frac{\partial P(\mathbf{Q}_{K_i^T})}{\partial q} q \\
&= A - \mathbf{Q}_{K_i^T} - c_{K_i}^T - q \\
&= A - c_{K_i}^T - \mathbf{Q}_{-i}^T - 2q
\end{aligned}$$

Using symmetry and $c_{K_i}^T = \frac{1}{K_i^T + 1}$, we have the following equilibrium condition;

$$2q_{K_i^T} = A - \frac{1}{K_i^T + 1} - \left(\sum_{K \neq K_i^T} \mathbf{N}(\mathbf{K}, \mathbf{T}) q_K + (\mathbf{N}(\mathbf{K}_i^T, \mathbf{T}) - 1) q_{K_i^T} \right)$$

That is,

$$(\mathbf{N}(\mathbf{K}_i^T, \mathbf{T}) + 1) q_{K_i^T} = A - \frac{1}{K_i^T + 1} - \sum_{K \neq K_i^T} \mathbf{N}(\mathbf{K}, \mathbf{T}) q_K$$

where $K, K_i^T \in \{K_{os}^T, \dots, T + 2\}$.

We have $q_K = q_{K'} + \frac{1}{K'+1} - \frac{1}{K+1}$ for any $K, K' \in \{K_{os}^T, \dots, T + 2\}$. To see this, note that

$$(\mathbf{N}(\mathbf{K}, \mathbf{T}) + 1) q_K = A - \frac{1}{K + 1} - \sum_{k \neq K} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k = A - \frac{1}{K + 1} - \mathbf{N}(\mathbf{K}', \mathbf{T}) q_{K'} - \sum_{k \neq K, K'} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k$$

and similarly,

$$(\mathbf{N}(\mathbf{K}', \mathbf{T}) + 1) q_{K'} = A - \frac{1}{K' + 1} - \sum_{k \neq K'} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k = A - \frac{1}{K' + 1} - \mathbf{N}(\mathbf{K}, \mathbf{T}) q_K - \sum_{k \neq K, K'} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k$$

Subtracting the two equations we get,

$$(\mathbf{N}(\mathbf{K}, \mathbf{T}) + 1) q_K - (\mathbf{N}(\mathbf{K}', \mathbf{T}) + 1) q_{K'} = \frac{1}{K' + 1} - \frac{1}{K + 1} + \mathbf{N}(\mathbf{K}, \mathbf{T}) q_K - \mathbf{N}(\mathbf{K}', \mathbf{T}) q_{K'}$$

That is,

$$q_K - q_{K'} = \frac{1}{K' + 1} - \frac{1}{K + 1}$$

Using this equality, we get the equilibrium quantity at K_{os}^T ,

$$(\mathbf{N}(\mathbf{K}_{os}^T, \mathbf{T}) + 1)q_{K_{os}^T} = A - \frac{1}{K_{os}^T + 1} - \sum_{K \neq K_{os}^T} \mathbf{N}(\mathbf{K}, \mathbf{T})q_K$$

$$(\mathbf{N}(\mathbf{K}_{os}^T, \mathbf{T}) + 1)q_{K_{os}^T} = A - \frac{1}{K_{os}^T + 1} - \sum_{K \neq K_{os}^T} \mathbf{N}(\mathbf{K}, \mathbf{T})\left(q_{K_{os}^T} + \frac{1}{K_{os}^T + 1} - \frac{1}{K + 1}\right)$$

$$(M + 1)q_{K_{os}^T} = A - \frac{1}{K_{os}^T + 1} - \sum_{K \neq K_{os}^T} \mathbf{N}(\mathbf{K}, \mathbf{T})\left(\frac{1}{K_{os}^T + 1} - \frac{1}{K + 1}\right)$$

$$q_{K_{os}^T} = \frac{A}{M + 1} - \frac{1}{(M + 1)(K_{os}^T + 1)} - \frac{1}{(M + 1)} \sum_{K \neq K_{os}^T} \mathbf{N}(\mathbf{K}, \mathbf{T}) \frac{K - K_{os}^T}{(K_{os}^T + 1)(K + 1)}$$

$$q_{K_{os}^T} = \frac{A}{M + 1} - \frac{1}{(M + 1)(K_{os}^T + 1)} - \frac{1}{(M + 1)} \sum_{t=1}^{T+2-K_{os}^T} \mathbf{N}(\mathbf{K}_{os}^T + \mathbf{t}, \mathbf{T}) \frac{t}{(K_{os}^T + 1)(K_{os}^T + t + 1)}$$

and

$$q_K = q_{K_{os}^T} + \frac{1}{K_{os}^T + 1} - \frac{1}{K + 1}$$

for each $K > K_{os}^T$.

Now we calculate the equilibrium profit levels. First, note that

$$(\mathbf{N}(\mathbf{K}_i^T, \mathbf{T}) + 1)q_{K_i^T} = A - \frac{1}{K_i^T + 1} - \sum_{K \neq K_i^T} \mathbf{N}(\mathbf{K}, \mathbf{T})q_K$$

where $K, K_i^T \in \{K_{os}^T, \dots, T + 2\}$. That is, in the equilibrium

$$q_{K_i^T} = A - \frac{1}{K_i^T + 1} - \sum_{K \neq K_i^T} \mathbf{N}(\mathbf{K}, \mathbf{T})q_{K^T} - \mathbf{N}(\mathbf{K}_i^T, \mathbf{T})q_{K_i^T} = A - \frac{1}{K_i^T + 1} - \sum_K \mathbf{N}(\mathbf{K}, \mathbf{T})q_K$$

Since

$$\mathbf{Q}_K = \sum_{T+2 \geq k \geq K_{os}^T} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k$$

and

$$\mathbf{P}_K = A - \mathbf{Q}_K = A - \sum_{T+2 \geq k \geq K_{os}^T} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k$$

we get

$$\begin{aligned} E[\pi_{K_i}^T] &= (\mathbf{P}_{K_i^T} - c_{K_i}^T) q_{K_i^T} \\ &= \left(A - \frac{1}{K_i^T + 1} + \sum_{T+2 \geq k \geq K_{os}^T} \mathbf{N}(\mathbf{k}, \mathbf{T}) q_k \right) q_{K_i^T} \\ &= (q_{K_i^T})^2 \end{aligned}$$

7.2 Proofs

Proof of Proposition 1.

We first show this result for $k_i^{T-1} = k_{os}^{T-1}$. Then, for $k_i^{T-1} > k_{os}^{T-1}$, it will be straightforward. If a firm with $k_i^{T-1} = k_{os}^{T-1}$ chooses not to use the open source, at $T - 1$, then the firm will have the following expected payoff.

$$\begin{aligned} V(0, k_{os}^{T-1}) &= p(0, k_{os}^{T-1}) [E[\pi_{k_{os}+1}^{T-1}] + \delta W_S(0, k_{os}^{T-1} + 1)] \\ &+ (1 - p(0, k_{os}^{T-1})) [E[\pi_{k_{os}}^{T-1}] + \delta W_F(0, k_{os}^{T-1})] - C(p(0, k_{os}^{T-1})) \\ &= p(0, k_{os}^{T-1}) [E[\pi_{k_{os}+1}^{T-1}] - E[\pi_{k_{os}}^{T-1}]] + E[\pi_{k_{os}}^{T-1}] \\ &+ p(0, k_{os}^{T-1}) [\delta W_S(0, k_{os}^{T-1} + 1) - \delta W_F(0, k_{os}^{T-1})] + \delta W_F(0, k_{os}^{T-1}) - C(p(0, k_{os}^{T-1})) \end{aligned}$$

Thus,

$$\begin{aligned} V(0, k_{os}^{T-1}) &= p(0, k_{os}^{T-1}) [E[\pi_{k_{os}+1}^{T-1}] - E[\pi_{k_{os}}^{T-1}] + \delta[W_S(0, k_{os}^{T-1} + 1) - W_F(0, k_{os}^{T-1})]] \\ &+ E[\pi_{k_{os}}^{T-1}] + \delta W_F(0, k_{os}^{T-1}) - C(p(0, k_{os}^{T-1})) \end{aligned}$$

Similarly, a firm with $k_i^{T-1} = k_{os}^{T-1}$ chooses to use the open source, at $T - 1$, then the firm will have the following expected payoff.

$$\begin{aligned} V(1, k_{os}^{T-1}) &= p(1, k_{os}^{T-1}) [E[\pi_{k_{os}+1}^{T-1}] - E[\pi_{k_{os}}^{T-1}] + \delta[W_S(1, k_{os}^{T-1} + 1) - W_F(1, k_{os}^{T-1})]] \\ &+ E[\pi_{k_{os}}^{T-1}] + \delta W_F(1, k_{os}^{T-1}) - C(p(1, k_{os}^{T-1})) \end{aligned}$$

Recall that the optimal investment levels are

$$C'(p(d_i^{T-1}, k_i^{T-1})) = E[\pi_{\kappa_i+1}^{T-1}] - E[\pi_{\kappa_i}^{T-1}] + \delta [W_S(d_i^{T-1}, \kappa_i^{T-1} + 1) - W_F(d_i^{T-1}, \kappa_i^{T-1})]$$

That is,

$$C'(p(0, k_{os}^{T-1})) = E[\pi_{k_{os}+1}^{T-1}] - E[\pi_{k_{os}}^{T-1}] + \delta [W_S(0, k_{os}^{T-1} + 1) - W_F(0, k_{os}^{T-1})]$$

$$C'(p(1, k_{os}^{T-1})) = E[\pi_{k_{os}+1}^{T-1}] - E[\pi_{k_{os}}^{T-1}] + \delta [W_S(1, k_{os}^{T-1} + 1) - W_F(1, k_{os}^{T-1})]$$

Thus,

$$V(0, k_{os}^{T-1}) = p(0, k_{os}^{T-1})C'(p(0, k_{os}^{T-1})) - C(p(0, k_{os}^{T-1})) + E[\pi_{k_{os}}^{T-1}] + \delta W_F(0, k_{os}^{T-1})$$

and

$$V(1, k_{os}^{T-1}) = p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) - C(p(1, k_{os}^{T-1})) + E[\pi_{k_{os}}^{T-1}] + \delta W_F(1, k_{os}^{T-1})$$

Note also that $W_F(0, k_{os}^{T-1}) = W_F(1, k_{os}^{T-1})$. Then, we get

$$\begin{aligned} V(0, k_{os}^{T-1}) - V(1, k_{os}^{T-1}) &= p(0, k_{os}^{T-1})C'(p(0, k_{os}^{T-1})) - C(p(0, k_{os}^{T-1})) \\ &\quad - [p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) - C(p(1, k_{os}^{T-1}))] \end{aligned}$$

Note that, the function $F(p) = pC'(p) - C(p)$ is an increasing function. To see this, look at the first derivative: $F'(p) = C'(p) + pC''(p) - C'(p) = pC''(p) > 0$ since $C'' > 0$ and $p > 0$. Thus, if $p(0, k_{os}^{T-1}) > p(1, k_{os}^{T-1})$, then $V(0, k_{os}^{T-1}) > V(1, k_{os}^{T-1})$.

Note that

$$C'(p(0, k_{os}^{T-1})) - C'(p(1, k_{os}^{T-1})) = \delta [W_S(0, k_{os}^{T-1} + 1) - W_S(1, k_{os}^{T-1} + 1)]$$

Here we have $W_S(0, k_{os}^{T-1} + 1) > W_S(1, k_{os}^{T-1} + 1)$. To see this, note that in period T , if firm i has used the open source in period $T - 1$, then it will be at the same technology level as the user firms by Lemma 1, because its success in period $T - 1$ will be public for each user firm at T . Denote the expected payoff of this case by W^{eq} . That is, $W^{eq} = W_S(1, k_{os}^{T-1} + 1)$. However, if firm i has not used the open source in period $T - 1$, then its technology level will be the same as all the user firms with probability ν and will be one step higher than all the user firms with probability $1 - \nu$, where ν is the probability that at least one user firm is successful in period $T - 1$. Denote the latter case's expected payoff with W^{sup} . Then, $W_S(0, k_{os}^{T-1} + 1) > W_S(1, k_{os}^{T-1} + 1)$ if $(1 - \nu)W^{sup} + \nu W^{eq} > W^{eq}$. However, $W^{sup} > W^{eq}$ since in the former expected payoff firm i has a higher technology level than the latter. Thus, $W_S(0, k_{os}^{T-1} + 1) > W_S(1, k_{os}^{T-1} + 1)$, which in turn implies

that $C'(p(0, k_{os}^{T-1})) > C'(p(1, k_{os}^{T-1}))$. Since $C'' > 0$, we get $p(0, k_{os}^{T-1}) > p(1, k_{os}^{T-1})$, which proves $V(0, k_{os}^{T-1}) > V(1, k_{os}^{T-1})$. When $k_i^{T-1} > k_{os}^{T-1}$, the proof works just the same. ■

Proof of Proposition 2.

If a firm with $k_i^{T-1} = k_{os}^{T-1} - 1$ chooses not to use the open source, at $T - 1$, then the firm will have the following expected payoff.

$$\begin{aligned}
V(0, k_{os}^{T-1} - 1) &= p(0, k_{os}^{T-1} - 1) [E[\pi_{k_{os}}^{T-1}] + \delta W_S(0, k_{os}^{T-1})] \\
&+ (1 - p(0, k_{os}^{T-1} - 1)) [E[\pi_{k_{os}-1}^{T-1}] + \delta W_F(0, k_{os}^{T-1} - 1)] \\
&- C(p(0, k_{os}^{T-1} - 1)) \\
&= p(0, k_{os}^{T-1} - 1) [E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}]] + E[\pi_{k_{os}-1}^{T-1}] \\
&+ p(0, k_{os}^{T-1} - 1) [\delta W_S(0, k_{os}^{T-1}) - \delta W_F(0, k_{os}^{T-1} - 1)] + \delta W_F(0, k_{os}^{T-1} - 1) \\
&- C(p(0, k_{os}^{T-1} - 1)) \\
&= p(0, k_{os}^{T-1} - 1) [E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] + \delta W_S(0, k_{os}^{T-1}) - \delta W_F(0, k_{os}^{T-1} - 1)] \\
&+ E[\pi_{k_{os}-1}^{T-1}] + \delta W_F(0, k_{os}^{T-1} - 1) - C(p(0, k_{os}^{T-1} - 1))
\end{aligned}$$

Recall the equilibrium investment probability of firm i with k_i^{T-1} and $d_i^{T-1} = 0$ is given by

$$C'(p(0, k_{os}^{T-1} - 1)) = E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] + [\delta W_S(0, k_{os}^{T-1}) - \delta W_F(0, k_{os}^{T-1} - 1)]$$

Plugging this we get

$$\begin{aligned}
V(0, k_{os}^{T-1} - 1) &= p(0, k_{os}^{T-1} - 1) C'(p(0, k_{os}^{T-1} - 1)) - C(p(0, k_{os}^{T-1} - 1)) \\
&+ E[\pi_{k_{os}-1}^{T-1}] + \delta W_F(0, k_{os}^{T-1} - 1)
\end{aligned}$$

Similarly we get

$$\begin{aligned} V(1, k_{os}^{T-1} - 1) &= p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) - C(p(1, k_{os}^{T-1})) \\ &+ E[\pi_{k_{os}}^{T-1}] + \delta W_F(1, k_{os}^{T-1}) \end{aligned}$$

Note that $W_F(1, k_{os}^{T-1}) = W_F(0, k_{os}^{T-1} - 1)$, since in both cases the firm enters the second stage of the last period at the same technology level by Lemma 1, thus will have the same expected period T payoff. Thus, we get

$$\begin{aligned} V(1, k_{os}^{T-1} - 1) - V(0, k_{os}^{T-1} - 1) &= p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) - C(p(1, k_{os}^{T-1})) \\ &- [p(0, k_{os}^{T-1} - 1)C'(p(0, k_{os}^{T-1} - 1)) - C(p(0, k_{os}^{T-1} - 1))] \\ &+ E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] \end{aligned}$$

Note that $E[\pi_{k_{os}-1}^{T-1}] = q_{k_{os}^{T-1}-1}^2$ and $E[\pi_{k_{os}}^{T-1}] = q_{k_{os}}^2$ and $q_{k_{os}^{T-1}-1} < q_{k_{os}}$. Thus, $E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] > 0$.

If $p(1, k_{os}^{T-1}) > p(0, k_{os}^{T-1} - 1)$, then, since $pC'(p) - C(p)$ is an increasing function,

$$p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) - C(p(1, k_{os}^{T-1})) > p(0, k_{os}^{T-1} - 1)C'(p(0, k_{os}^{T-1} - 1)) - C(p(0, k_{os}^{T-1} - 1)).$$

This, together with $E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] > 0$, implies $V(1, k_{os}^{T-1} - 1) > V(0, k_{os}^{T-1} - 1)$.

If, however, $p(1, k_{os}^{T-1}) < p(0, k_{os}^{T-1} - 1)$, then we have

$$\begin{aligned}
V(1, k_{os}^{T-1} - 1) - V(0, k_{os}^{T-1} - 1) &= p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) - C(p(1, k_{os}^{T-1})) \\
&- [p(0, k_{os}^{T-1} - 1)C'(p(0, k_{os}^{T-1} - 1)) - C(p(0, k_{os}^{T-1} - 1))] \\
&+ E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] \\
&= C(p(0, k_{os}^{T-1} - 1)) - C(p(1, k_{os}^{T-1})) \\
&+ p(1, k_{os}^{T-1})C'(p(1, k_{os}^{T-1})) \\
&- p(0, k_{os}^{T-1} - 1)C'(p(0, k_{os}^{T-1} - 1)) \\
&+ E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] \\
&> C(p(0, k_{os}^{T-1} - 1)) - C(p(1, k_{os}^{T-1})) \\
&- C'(p(0, k_{os}^{T-1} - 1)) + E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] \\
&= C(p(0, k_{os}^{T-1} - 1)) - C(p(1, k_{os}^{T-1})) \\
&- [E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] + \delta W_S(0, k_{os}^{T-1}) - \delta W_F(0, k_{os}^{T-1} - 1)] \\
&+ E[\pi_{k_{os}}^{T-1}] - E[\pi_{k_{os}-1}^{T-1}] \\
&= C(p(0, k_{os}^{T-1} - 1)) - C(p(1, k_{os}^{T-1})) \\
&- [\delta W_S(0, k_{os}^{T-1}) - \delta W_F(0, k_{os}^{T-1} - 1)]
\end{aligned}$$

The inequality follows since $C' > 0$ and $p(0, k_{os}^{T-1} - 1)$ can be at most 1. Note that $C(p(0, k_{os}^{T-1} - 1)) - C(p(1, k_{os}^{T-1})) > 0$ since $C' > 0$. Also note that, $W_S(0, k_{os}^{T-1}) = W_F(0, k_{os}^{T-1} - 1)$. This is because at the end of the first stage of period T (after open source use decisions are made), the technology level of the firm will be the same under the both cases (again by Lemma 1), and since the firm is not using the open source in period $T - 1$, its own success at period $T - 1$ will not be available for the other firms in

period T , thus the two expected payoffs are the same. Thus,

$$V(1, k_{os}^{T-1} - 1) - V(0, k_{os}^{T-1} - 1) > C(p(0, k_{os}^{T-1} - 1)) - C(p(1, k_{os}^{T-1})) > 0$$

which completes the proof. ■