A Comparative Analysis of Alternative Life-Cycle Investment Strategies for Turkey

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Abstract

In this paper, we perform a welfare comparison of alternative life-cycle investment strategies for the pension funds using Monte Carlo simulations in an heterogeneous agent framework for Turkey. We calibrate model parameters based on historical data and compare a set of strategies provided by the pension fund industry and suggested by the asset management literature. Our results reveal that the glide path proposed by Bodie et al. [4] improves upon the alternative options in a default setting, but risk-averse individuals might hedge their risks by investing in housing.

Keywords: Life-cycle portfolio decisions, human capital, housing, stock market participation

1. Introduction

Established in 2003, the Turkish private pension system is, in accordance with the worldwide trend, based on defined contribution (DC) plans and targets to promote long-term savings [17]. Defined contribution plans delegate the responsibility of investment decisions to individuals such as participation in a plan, the amount and allocation of contributions, portfolio re-balancing and withdrawal of the accumulated sum at retirement. Even though DC plans provide flexibility to the individual investors, they pave the way to uninformed decisions and sub-optimal choices. These developments in the pension fund system necessitates the construction of investment strategies to guide participants for optimal allocation of funds. In order to address this issue, pension fund companies provide life-cycle or target-date funds which are characterized by a fixed retirement date and a glide path gradually reducing the allocation to risky assets as the target date approaches. However, there is no consensus on the optimal glide path on the allocation from risky to conservative assets over time as it depends on both the characteristics of the investor and the financial markets.

Turkish private pension fund system also provides target date funds which automatically adjust portfolio allocations according to the time left to retirement, however, to the best of our knowledge, there is not a comprehensive study comparing the performance of alternative life-cycle strategies which takes into account the risk aversion of individuals, properties of the labor and financial markets in Turkey. In this paper, we aim to fill this gap in the literature by providing a thorough

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comparison of investment strategies proposed by the pension fund industry and asset management literature through Monte Carlo simulations for Turkey.

The classical portfolio theory suggests that individuals should invest a fixed percentage of their funds in risky assets, depending on their risk tolerance [13]. However, the popular policy advice in the industry has been to aggressively invest in risky assets in the beginning of an investment cycle and to slowly decrease the share of risky assets in the portfolio, as the retirement approaches [11]. In contrast to the both advice, Chang et al. [7] observed that the actual stock share of a portfolio over the life-cycle exhibits a hump-shaped profile rather than a glide path, i.e. young investors are not aggressive and they slowly increase the share of risky assets in their portfolio until a certain age and then slowly decrease it, as their retirement approaches.

Bodie et al. [4] considers human capital, defined as the total wage earnings an individual expects until the retirement age, in the investment decision – the steady depletion of human capital with age explains the decrease in the ratio of risky assets over life. Furthermore, Flavin and Yamashita [9] finds that individuals who own a house tend to invest more aggressively than those who do not. This explains why young investors are not aggressive – they are investing in housing. These two opposing forces explain the hump-shaped investment profile observed by Chang et al. [7].

Chang et al. [6] and Pfau [19] compare welfare outcomes of alternative glide path investment strategies for the U.S. market and Olear et al. [16] compare investment strategies for different labor income profiles and sectors of work for investors in Netherlands. Similarly, in this paper, we perform a comparative analysis of alternative investment strategies for individual wage-earners in Turkey in an expected utility framework with heterogeneous agents. We consider the "fixed over the lifetime" investment strategies, proposed by Markowitz [12] and Merton [13] and "life-cycle glide path" heuristics proposed in the industry [8]. Finally, we consider the optimal glide path suggested by dynamic mean-variance problem that takes housing investment into account as proposed by Munk [14]. We use Monte Carlo simulations that rely on parameters calibrated for the Turkish economy. Furthermore, we allow for heterogeneity among agents on the dimensions such as sector of work, level of education and risk aversion which enables a thorough welfare comparison of agents with different demographic characteristics.

The remainder of the paper is organized as follows: Section 2 presents the theoretical framework we will use in our simulations and a detailed information on the data sources and calibration. Section 3 compares the simulation results of alternative life-cycle investment strategies and Section 4 concludes.

2. Data and Methodology

2.1. Portfolio allocation models

In our analysis, we compare different dynamic portfolio allocation strategies given in the literature. Markowitz [12] suggested the optimal share of stocks in a single-period portfolio would be represented by the following equation:

$$\alpha = \frac{\mu - R_f}{\gamma \sigma^2}, \tag{1}$$

where R_f is a risk-free rate, μ and σ^2 are mean and variance of rate of return on stocks, and γ is the parameter of relative risk aversion. Merton [13] generalized this problem to multiple periods using dynamic programming and found that it is optimal for all households to repeat the same fixed mean-variance solution every period.

However, these results are inconsistent with the popular financial advice suggesting that younger investors should have higher share of stocks in their portfolios, and older investors – higher share of risk-free assets. This advice is summarized by the well-known rule of thumb:

$$\alpha_t (100 - t)\%, \tag{2}$$

where t is the age of an investor and α_t is the portfolio equity share at age t.

Bodie et al. [4] added human capital into the Merton [13]'s dynamic model and found that for complete markets and risk-free labor income, the optimal share of stocks in a portfolio is given by a generalization of Markowitz [12]:

$$\alpha_t = \frac{\mu - R_f}{\gamma \sigma^2} \left(\frac{F_t + L_t}{F_t} \right), \tag{3}$$

where F_t and L_t are financial and human capital of an agent at age t respectively. Depletion of human capital over the life-cycle relative to the financial wealth produces an optimal investment strategy in the form of a glide path i.e. younger agents carry a higher share of stocks in their portfolio which gradually declines with age. Cocco et al. [8] extended this model for the case of stochastic labor income and obtain a recursive solution where share of stocks in a portfolio at age t could be approximated by the following rule of thumb:

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60] \\ 50\% & t > 60 \end{cases} \tag{4}$$

Finally, we consider the constrained optimal solution to dynamic mean-variance problem proposed by Munk [14]:

$$\boldsymbol{\alpha'_t} = \frac{1}{\gamma} \left[\left(\frac{F_t + L_t}{F_t} \right) \cdot (\boldsymbol{\mu'} + \lambda \cdot \mathbf{1'}) - \gamma \cdot \frac{L_t}{F_t} \cdot \mathbf{cov'}(\mathbf{r}, r_L) \right] \mathbf{\Sigma^{-1}}, \tag{5}$$

where λ is a scalar given by:

$$\lambda = \frac{\gamma + \left[\gamma \cdot \frac{L_t}{F_t} \cdot \mathbf{cov'}(\mathbf{r}, r_L) - (1 + \frac{L_t}{F_t}) \cdot \boldsymbol{\mu'} \right] \mathbf{\Sigma^{-1} \cdot 1}}{(\frac{F_t + L_t}{F_t}) \cdot (\mathbf{1' \Sigma^{-1} 1})} \tag{6}$$

See Appendix A for derivation and details.

2.2. Asset classes

We employ monthly data for the Turkish stock market, BIST100,[1] covering the period 2000–2020, and Turkish housing market, REIDIN[2], covering the period 2003–2020, to construct stock and house price series. Figures 1 and 2 show the time series charts for stock and housing prices, and Table 1 shows the descriptive statistics of monthly rates for stocks and housing, and maturity rates of 10-year bonds.

[1] BIST100 is an index for the stocks of 100 largest companies in Turkey.
[2] We used residential sales price index for Istanbul calculated using Laspeyres' formula.

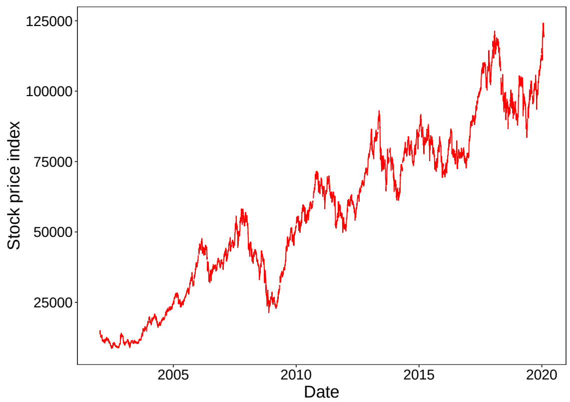


Figure 1: BIST100 Turkish stock market prices (2000-2020)

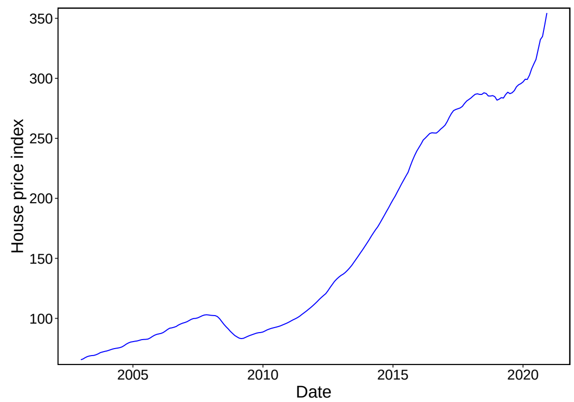


Figure 2: REIDIN Turkish house price index (2003-2020)

	BIST100	Housing	Bonds
Mean	0.00923	0.00781	0.10521
Median	0.00757	0.00799	0.0973
Std. Dev.	0.10415	0.00886	0.02795
Variance	0.01085	0.00008	0.00078
Skewness	0.03547	-1.18178	1.55354
Kurtosis	5.24545	5.71811	5.2521
Minimum	-0.43674	-0.0242	0.0617
Maximum	0.43279	0.02838	0.207

Table 1: Descriptive statistics for monthly returns for stock prices (2000-2020), housing prices (2003-2020), and annual maturity rates for 10-year bonds (2010-2020)

2.3. Labor income process

We construct labor income series using Household Labor Force Survey provided by the Turkish Statistical Institute's [10] which is in the form of a repeated cross-sections. In line with Aktuğ et al. [1], we construct a pseudo-panel with 55 thousand data points for 170 households-cohorts for 2002 to 2020. We deflate nominal wages with the consumer price index to obtain real wages. Figure 3 presents life-cycle income profile, which exhibits a hump-shaped pattern consistent with the literature.[3] Figure 4 shows the lifetime labor income series for different levels of education. Notice that the curves are hump-shaped over lifetime and are steeper for higher levels of education. We further refer to wages of individuals with undergraduate, high school, and primary education as "steep", "moderate", and "flat" wage dynamics, respectively.

Following Campbell et al. [5], we model labor income process as a function $f(t, d_i)$ of age t and the education level d_i plus idiosyncratic shocks ϵ_{it}.[4] Upon reaching the retirement age R, an individual receives a certain percentage λ of his/her last wage. The stochastic labor income process

[3] See for example Ben-Porath [3].

[4] We abstract from aggregate shocks, as they solely add an additional source of volatility without affecting our comparative analysis.

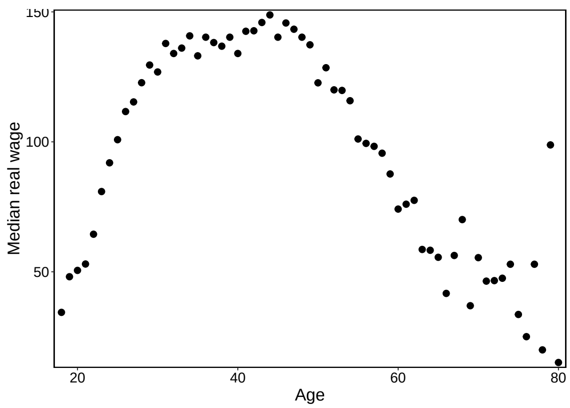


Figure 3: Median Turkish salaries by age

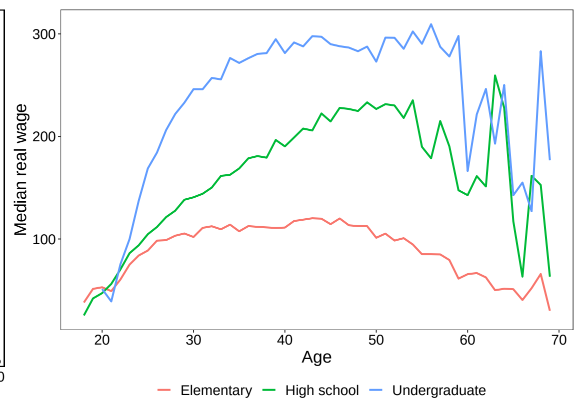


Figure 4: Lifetime real wage dynamics by education level.

is, thus, given as:

$$\log(Y_{i,t}) = \begin{cases} f(t, d_i), & t \leq R \\ f(R, d_i) + \log(\lambda), & t > R \end{cases} \tag{7}$$

In particular, we perform regressions[5] of wages on age and age squared for different education levels:

$$\log(Y_{it}) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 d_i^H + \beta_4 d_i^U + \epsilon_{it} \tag{8}$$

where d_i^H and d_i^U are dummy variables for high school and undergraduate education levels, and $t \leq R$.

2.4. Simulated time series

Using parameters estimated in Sections 2.2 and 2.3, we generate correlated time series for stock prices, house prices, and labor income. In line with Ascheberg et al. [2], we assume that these series follow geometric Brownian motion with drifts μ_S, μ_H, μ_L, volatilities σ_S, σ_H, σ_L, and pairwise correlations $\rho_{SH}, \rho_{HL}, \rho_{SL}$. [6]

The drift and volatility parameters for the geometric Brownian motion process governing the rate of return for stocks are estimated as $\mu_S = 11.72\%$ and $\sigma_S = 36.16\%$ respectively. Similarly, for the return process for house prices, drift and volatility are obtained as $\mu_H = 9.84\%$ and $\sigma_H = 3.07\%$. Risk-free rate is set to 12% according to the OECD [15] forecast, and, upon subtracting the medium-term inflation rate forecast 9% provided by the Turkish Central Bank, which gives an annual real interest rate equal to $r_f = 3\%$. The house-stock price and house-wage prices contemporaneous correlations are estimated as $\rho_{SH} = 0.145$ and $\rho_{HL} = 0.117$.

In generating labor income series, we take into account the heterogeneities in (i) education level and (ii) sectors of work. The former is characterized by different wage dynamics, as included in Equation 8. To capture the latter, we observe that wages in different sectors of work have different

[5] See Appendix C for details of the regression results.
[6] See Appendix B for details.

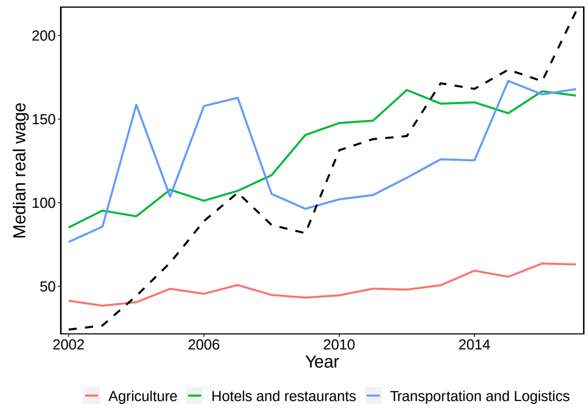


Figure 5: Historical real wage dynamics by sector. Dashed line shows the normalized stock price index.

correlations to the stock prices, ρ_{SL}. This heterogeneity is presented in Figure 5 that plots median real wages for different sectors, along with normalized stock prices. We use three measures of ρ_{SL} in our benchmark: 0, 0.2 and 0.4.

2.5. Human capital and Financial capital

To simulate life-cycle portfolios given by Equations 3, 5, and 6, we need to construct human capital L_t and financial capital F_t series. Human capital series is constructed as the discounted sum of all future wages until retirement with the discount factor r_f.

$$L_t = \sum_{\tau=t}^{R} \frac{Y_\tau}{(1+r_f)^{\tau-t}} \tag{9}$$

Figure 6 illustrates the evolution of the human capital for flat, moderate, and steep wages for different levels of age. In order to construct the evolution of the financial capital, we assume that agents invest a certain fraction c of their wage in the retirement portfolio every month ($c = 3\%$ is a default as per Pension Monitoring Center [18]). Previously invested amount accrues at the portfolio rate of return r_p. The accumulation of the financial capital is given by:

$$F_t = \sum_{\tau=0}^{t} c \cdot w_\tau \cdot (1+r_p)^{t-\tau} \tag{10}$$

Figure 7 demonstrates the evolution of a sample financial capital for a naive fixed investment strategy of 50% in stocks and 50% in bonds.

2.6. Monte Carlo Simulations of Life-cycle Portfolios

In accordance with Munk [14], we run our simulations for a 25-year-old agent who invests in her retirement for 40 years until she reaches retirement at the age of 65. We run 10,000 simulations for each of the 6 life-cycle portfolios mentioned in Section 2.1.

Moreover, combining the approaches of Olear et al. [16] and Munk [14], we consider three aspects of heterogeneity in agents: (i) 3 education levels, (ii) 3 sectors of work, and (iii) 4 risk aversion

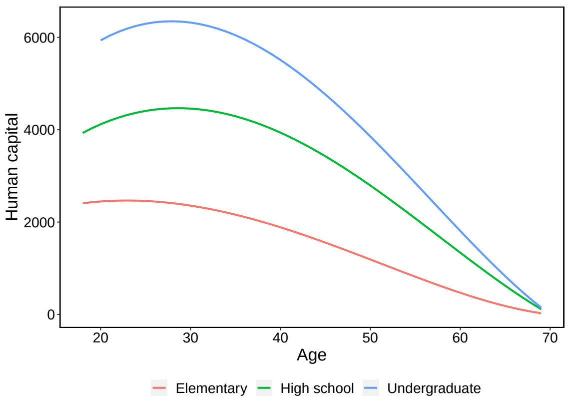


Figure 6: Human capital at every age for individuals with steep, moderate and flat wage growth curves. As individuals get older, their human capital gradually depletes.

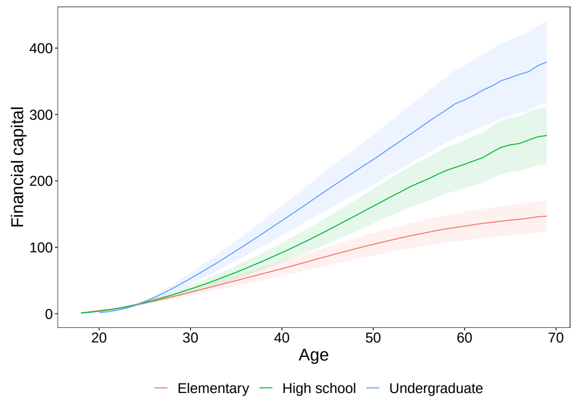


Figure 7: Financial wealth accumulation through life, by a naive agent who invests 50% in bonds and 50% in stocks (with 95% confidence intervals).

levels. The former two are described in Section 2.4. To analyze the latter, we re-run simulations for $\gamma \in \{1.5, 3, 5, 10\}$, with $\gamma = 1.5$ as default (as given for Turkey by Torul and Oztunali [20]).

Consequently, we consider 6 portfolios for each of the $3 \cdot 3 \cdot 4 = 36$ different agents, i.e. a total of 216 simulation scenarios. The first three options are industry practice such as $100 - age$, Cocco et al. [8] and Markowitz [12], given by Equations 2, 4 and 1 respectively. Figures 8 (a-c) illustrate stock-bond allocations for every age for these portfolios. Next three strategies are proposed by Bodie et al. [4] and Munk [14] and are given by Equations 3 and 5. These strategies propose younger investors to allocate all of their funds in stocks (or housing), and gradually decrease their share in the portfolio, as they get older. Note that these strategies require recalculation of portfolio allocation at every period, depending on the accumulated financial capital F_t, remaining human capital H_t, and the risk aversion coefficient γ of an agent. Figures 8 (d-f) illustrate sample portfolio allocations for these strategies. [7].

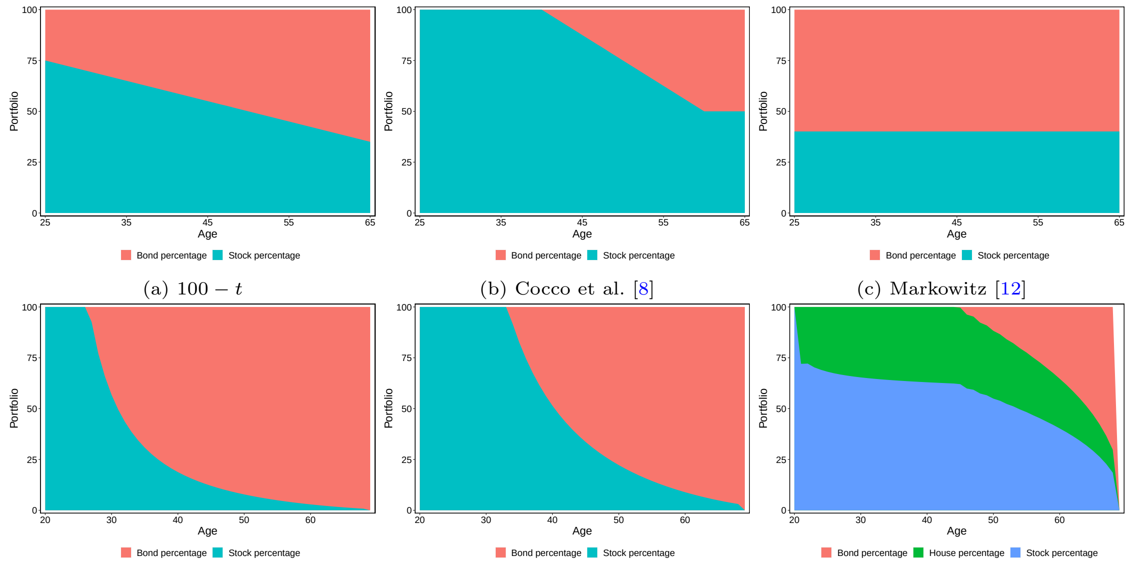
3. Results

In this section, we provide a comparative analysis of alternative life-cycle strategies in terms of capital accumulated at the time of retirement and the welfare measured by the mean-variance utility. We present results from Monte-Carlo simulations with 10,000 repetitions for each scenario described in Section 2.6.

3.1. Accumulated wealth

First of all, we compare the accumulated capital W_{65} at the retirement for six different investment strategies described in Section 2. We use the benchmark scenario with moderate correlation of wages to stock prices ($\rho_{SL} = 0.2$) and default risk aversion ($\gamma = 1.5$). Figure 9 presents the box plots for moderate labor income process and Table 3 shows the mean and variance of W_{65} for each portfolio for flat, moderate, and steep labor income curves. We find that the dynamic portfolio by

[7]See Appendix D for a details



(a) $100 - t$ (b) Cocco et al. [8] (c) Markowitz [12]

(d) Bodie et al. [4] (e) Munk [14] without housing (f) Munk [14] with housing

Figure 8: Portfolio allocations at every age, proposed by alternative life-cycle strategies. The optimal allocations at every age for (d-f) depend on the previous realizations of volatile stock returns, individual risk-aversion, and wage curves. See Appendix D for details.

Bodie et al. [4] accumulates the least capital, while the glide path strategies given by Cocco et al. [8] and Munk [14] accumulate more capital than other strategies. This finding is robust for different sectors of work (defined by correlation of wages to stock prices)[8].

Next, we look at the effects of risk aversion on the investment outcomes. Table 4 shows the mean and variance of all six investment strategies for highly risk-averse individuals ($\gamma = 10$). We find that while capital accumulated is unaffected by the risk aversion level, the optimal glide path strategies by Bodie et al. [4] and Munk [14] result in less volatility for risk-averse individuals in expense of minor decrease in the expected value.

3.2. Welfare comparison

Welfare of individuals is measured by the mean-variance utility that (a) takes risk aversion into account and (b) addresses the trade-off between returns and volatility:

$$U(W_{65}) = E[W_{65}] - \frac{\gamma}{2}\mathrm{var}(W_{65}) \tag{11}$$

First of all, we calculate the utilities for the six investment strategies in a benchmark scenario with moderate stock-income correlation ($\rho_{SL} = 0.2$) and the default risk aversion ($\gamma = 1.5$), presented in Table 5. We find that the strategies by Markowitz [12] and Bodie et al. [4] are more preferable than other options. Particularly, we see that the strategy by Munk [14] with and without housing perform worst in terms of welfare. We find, however, that the utility of the investment

[8]See Appendix E for the sensitivity analysis

Parameter	Description	Value
Y	Beginning age	25
R	Retirement age	65
T	Lifespan (years)	100
γ	Risk aversion	$[1.5, 5, 10, 20]$
β	Discount rate	0.89
r_f	Risk-free rate	0.03
c	Share of wage invested	0.03
μ_S	Expected stock returns	0.1172
μ_H	Expected housing returns	0.0984
σ_S	Stock returns volatility	0.3616
σ_H	Housing returns volatility	0.0307
ρ_{SH}	House-stock correlation	0.145
ρ_{HL}	House-wage correlation	0.1172
ρ_{SL}	House-wage correlation	$[0, 0.2, 0.4]$

Table 2: Benchmark Parameters

	Markowitz	100-age	Cocco et al.	Bodie et al.	Munk	Munk (housing)
Elementary	132.075	133.491	136.646	128.236	136.461	136.054
school	(3.628)	(4.622)	(6.720)	(3.067)	(7.444)	(6.835)
High school	246.684	248.723	254.230	237.123	252.489	251.731
	(6.726)	(7.948)	(11.455)	(5.032)	(13.747)	(12.317)
Undergraduate	353.242	356.457	364.511	340.494	362.530	361.425
	(9.572)	(11.624)	(16.797)	(7.527)	(19.579)	(17.670)

Table 3: Mean and standard deviation of accumulated capital W_{65} for 6 investment strategies for flat, moderate, and steep income curves with moderate stock market correlation ($\rho_{SL} = 0.2$) and default level of risk aversion ($\gamma = 1.5$).

	Markowitz	100-age	Cocco et al.	Bodie et al.	Munk	Munk (housing)
Elementary	132.095	133.527	136.728	126.057	133.311	133.691
school	(3.614)	(4.588)	(6.677)	(1.594)	(5.939)	(1.495)
High school	246.708	248.776	254.353	233.203	246.819	247.271
	(6.737)	(7.928)	(11.432)	(2.853)	(10.524)	(2.765)
Undergraduate	353.279	356.535	364.694	334.816	354.273	355.062
	(9.566)	(11.569)	(16.729)	(4.127)	(15.179)	(3.938)

Table 4: Mean and standard deviation of accumulated capital W_{65} for 6 investment strategies for flat, moderate, and steep income curves with moderate stock market correlation ($\rho_{SL} = 0.2$) and high level of risk aversion ($\gamma = 10$).

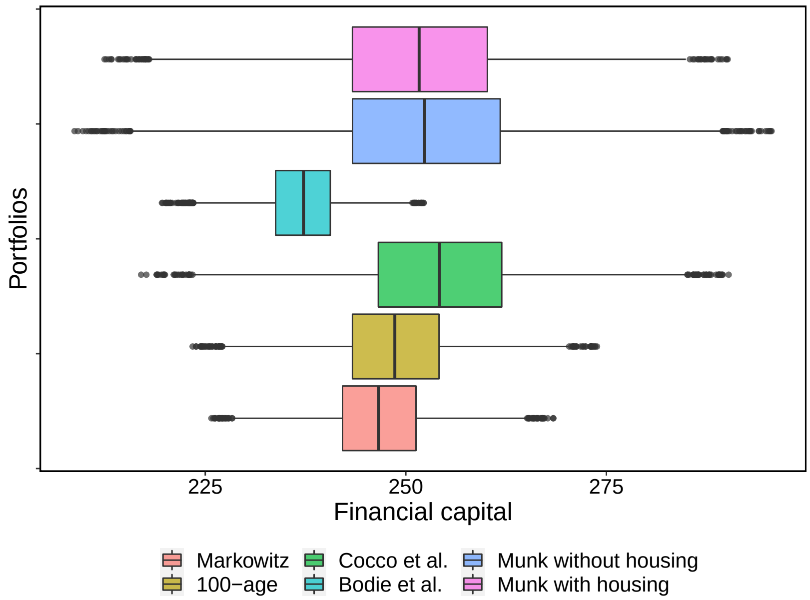


Figure 9: Accumulated capital W_{65} for 6 investment strategies for the moderate income curve (high school) and stock market correlation ($\rho_{SL} = 0.2$), and the default risk aversion level ($\gamma = 1.5$).

	Markowitz	100-age	Cocco et al.	Bodie et al.	Munk	Munk (housing)
Elementary	122.201	117.468	102.781	121.183	94.899	101.0111
High school	212.756	201.349	155.819	218.134	110.751	137.946
Undergraduate	284.523	255.124	152.918	298.007	75.0135	127.265

Table 5: Utilities $U(W_{65})$ for 6 investment strategies for flat, moderate, and steep income curves with moderate stock market correlation ($\rho_{SL} = 0.2$) and default level of risk aversion ($\gamma = 1.5$).

strategies except for Munk [14] with housing drops with the risk aversion level, as shown in the Figure 10. Moreover, the strategy by Munk [14] with housing is the most preferable strategy for $\gamma \geq 5$. This finding is more pronounced for sectors highly correlated to the stock market ($\rho_{SL} = 0.4$) (see Appendix E).

Finally, we look at the effect of the sector of work (stock-income correlation) on the utilities of the investment strategies. Table 6. Table 6 shows the utilities for the 6 investment strategies for individuals working in the sector highly correlated to the stock market ($\rho = 0.4$) with the moderate level of risk aversion ($\gamma = 3$). We can see that the strategy by Munk [14] with housing is more preferable to others, including Bodie et al. [4]. Figure 11 clarifies this result by showing that the welfare of Munk [14] with housing increases with the stock-income correlation, while it is unaffected for the other investment strategies. See Appendix E for more details.

4. Conclusion

In this paper, we present a comparative analysis of alternative life-cycle investment strategies for Turkey using Monte Carlo simulations. Parameters used in the simulations are calibrated/estimated

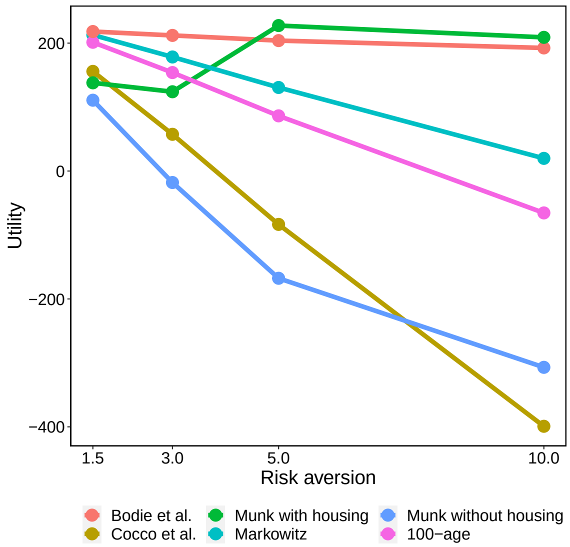


Figure 10: Utility $U(W_{65})$ resulting from the 6 investment strategies for 4 levels of risk aversion.

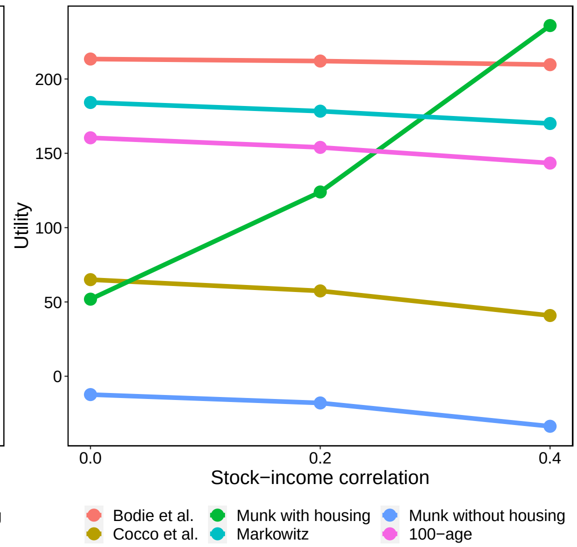


Figure 11: Utility $U(W_{65})$ resulting from the 6 investment strategies for 3 levels of stock-income correlation.

	Markowitz	100-age	Cocco et al.	Bodie et al.	Munk	Munk (housing)
Elementary	94.720	73.9025	13.544	114.677	9.722	127.966
High school	118.464	71.692	-104.932	202.410	-156.448	227.801
Undergraduate	92.734	-22.410	-407.973	264.141	-479.025	315.364

Table 6: Utilities $U(W_{65})$ for 6 investment strategies for flat, moderate, and steep income curves with moderate stock market correlation ($\rho_{SL} = 0.4$) and moderate level of risk aversion ($\gamma = 3$).

to reflect the properties of the Turkish economy and financial markets. In our simulations, we account for differences in the level of education, sector of work and risk aversion levels. We calculate the accumulated wealth and the resulting utilities for the selected strategies which are either proposed by the pension fund industry in the form of heuristics or provided as optimal solutions obtained in a dynamic optimization framework. Our results reveal that, the strategy proposed by Bodie et al. [4] is preferable to other strategies in default settings. Furthermore, we show that risk averse agents can greatly improve their welfare by investing in housing based on the strategy provided by Munk [14].

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Appendix A. Derivation of Munk's Constrained Mean-Variance Solution

The dynamic mean-variance problem is created by replacing static wealth W_t with dynamic ratio $\left(\frac{W_{t+1}}{W_t}\right)$. The problem is:

$$\max\left\{E\left[\frac{W_{t+1}}{W_t}\right]-\frac{\gamma}{2}\text{var}\left(\frac{W_{t+1}}{W_t}\right)\right\} \tag{A.1}$$

We also add the human capital to the model:

$$W_t = F_t + L_t \tag{A.2}$$

Thus, the law of motion is:

$$W_{t+1} = F_t\left(1+\boldsymbol{\pi}'\mathbf{r}\right) + L_t\left(1+r_L\right) \tag{A.3}$$

where $\boldsymbol{\pi}$ is a vector of shares of assets in a portfolio, $\mathbf{r}$ are rates of return on assets, and r_L is the rate of return on human capital, such that $\mathbf{r} \sim (\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $r_L \sim (\mu_L, \sigma_L)$.

Thus, the moments become:

$$E\left[\frac{W_{t+1}}{W_t}\right] = \frac{1}{1+l}\left(1+\boldsymbol{\pi}'\boldsymbol{\mu}\right) + \frac{l}{1+l}\left(1+\mu_L\right) \tag{A.4}$$

$$\text{var}\left(\frac{W_{t+1}}{W_t}\right) = \left(\frac{1}{1+l}\right)^2 \boldsymbol{\pi}'\boldsymbol{\Sigma}\boldsymbol{\pi} + \left(\frac{l}{1+l}\right)^2 \sigma_L^2 + 2\frac{l}{(1+l)^2}\cdot\boldsymbol{\pi}'\mathbf{cov}(\mathbf{r}, r_L) \tag{A.5}$$

where $l = \frac{L_t}{F_t}$. Thus, we can rewrite Equation A.1 as:

$$\max_{\boldsymbol{\pi}}\left\{\frac{1}{1+l}\left(\left(1+\boldsymbol{\pi}'\boldsymbol{\mu}\right)+l\left(1+\mu_L\right)\right) - \frac{\gamma}{2(1+l)^2}\cdot\left(\boldsymbol{\pi}'\boldsymbol{\Sigma}\boldsymbol{\pi} + l^2\sigma_L^2 + 2l\cdot\boldsymbol{\pi}'\mathbf{cov}(\mathbf{r}, r_L)\right)\right\} \tag{A.6}$$

Cancelling out redundant elements, our optimization problem takes the following form:

$$\max_{\boldsymbol{\pi}}\left\{\boldsymbol{\pi}'\boldsymbol{\mu} - \frac{\gamma}{2(1+l)}\cdot\left(\boldsymbol{\pi}'\boldsymbol{\Sigma}\boldsymbol{\pi} + 2l\cdot\boldsymbol{\pi}'\mathbf{cov}(\mathbf{r}, r_L)\right)\right\} \tag{A.7}$$

subject to:

$$\pi'1 = 1 \tag{A.8}$$

$$-\pi_i \leq 0, \forall i \tag{A.9}$$

The Lagrangean is:

$$L = \boldsymbol{\pi}'\boldsymbol{\mu} - \frac{\gamma}{2(1+l)}\cdot\left(\boldsymbol{\pi}'\boldsymbol{\Sigma}\boldsymbol{\pi} + 2l\cdot\boldsymbol{\pi}'\mathbf{cov}(\mathbf{r}, r_L)\right) + \lambda\cdot\left(\boldsymbol{\pi}'\mathbf{1} - 1\right) - \boldsymbol{\psi}'\boldsymbol{\pi}, \tag{A.10}$$

where ψ is a vector of Lagrange coefficients for non-negativity constraints for all π_i. Karush-Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial \boldsymbol{\pi}}_{(\psi_i \geq 0)} \geq 0 \tag{A.11}$$

$$\pi_i \cdot \frac{\partial L}{\partial \pi_i}_{(\psi_i = 0)} = 0 \tag{A.12}$$

$$\boldsymbol{\pi'1} = 1 \tag{A.13}$$

$$\psi_i \cdot \pi_i = 0 \tag{A.14}$$

$$\pi_i, \psi_i, \lambda \geq 0 \tag{A.15}$$

for all i. Equation A.11 yields:

$$\boldsymbol{\mu'} - \frac{\gamma}{2(1+l)} \cdot (2\boldsymbol{\pi'\Sigma} + 2l \cdot \mathbf{cov}'(\mathbf{r}, r_L)) + \lambda \cdot \boldsymbol{1'} - \boldsymbol{\psi'} = 0 \tag{A.16}$$

which is equivalent to:

$$\boldsymbol{\pi'} = \frac{1}{\gamma} \left[(1+l) \cdot (\boldsymbol{\mu'} + \lambda \cdot \boldsymbol{1'} - \boldsymbol{\psi'}) - \gamma \cdot l \cdot \mathbf{cov}'(\mathbf{r}, r_L) \right] \boldsymbol{\Sigma^{-1}} \tag{A.17}$$

Substituting this into Equation A.13 we obtain:

$$\boldsymbol{\pi'1} = \frac{1}{\gamma} \left[(1+l) \cdot (\boldsymbol{\mu'} + \lambda \cdot \boldsymbol{1'} - \boldsymbol{\psi'}) - \gamma \cdot l \cdot \mathbf{cov}'(\mathbf{r}, r_L) \right] \boldsymbol{\Sigma^{-1}1} = 1 \tag{A.18}$$

which yields:

$$\lambda = \frac{\gamma + \left[\gamma \cdot l \cdot \mathbf{cov}'(\mathbf{r}, r_L) + (1+l) \cdot \boldsymbol{\psi'} - (1+l) \cdot \boldsymbol{\mu'} \right] \boldsymbol{\Sigma^{-1} \cdot 1}}{(1+l) \cdot (\boldsymbol{1'\Sigma^{-1}1})} \tag{A.19}$$

When $\pi_i > 0$ we use the Equations A.17 and A.18 with $\psi_i = 0$. Otherwise, $\psi_i > 0$ and:

$$\frac{\partial L}{\partial \pi_i}_{(\psi_i = 0)} > \frac{\partial L}{\partial \pi_i}_{(\psi_i > 0)} = 0, \tag{A.20}$$

which, combined with Condition A.12 forces $\pi_i = 0$. Next, we reduce the dimension of the problem and solve for nonnegative values.

Appendix B. Ascheberg's correlation structure

The structure that returns desired correlation coefficients ρ_{SL}, ρ_{SH} and ρ_{HL} is as follows:

$$\frac{\Delta S_{t+1}}{S_t} = \mu_S + \sigma_S \cdot \epsilon_{St} \tag{B.1}$$

$$\frac{\Delta H_{t+1}}{H_t} = \mu_H + \sigma_H \cdot \left(\epsilon_{St}\rho_{SH} + \epsilon_{Ht}\sqrt{1-\rho_{SL}^2} \right) \tag{B.2}$$

$$\frac{\Delta Y_{t+1}}{Y_t} = \mu_L + \sigma_L \cdot \left(\epsilon_{St}\rho_{SL} + \epsilon_{Ht}\frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}} + \epsilon_{Lt}\sqrt{1-\rho_{SL}^2-(\frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}})^2} \right) \tag{B.3}$$

To derive this, let Σ be a correlation matrix of a vector $X = (x_1, x_2, ..., x_K)$. Also, let $\Sigma = LL'$ be a Cholesky decomposition of this matrix.

Notice that the variance-covariance matrix of an i.i.d. random vector $\Omega = (\epsilon_1, \epsilon_2, ..., \epsilon_K)$ with variances equal to 1, is an identity matrix. Thus, the product $L\Omega$ has the same correlation structure as X:

$$\text{cov}(L\Omega) = E[(L\Omega)(L\Omega)'] = E[L\Omega\Omega' L'] = L \cdot E[\Omega\Omega'] \cdot L' = L \cdot \text{var}(\Omega) \cdot L' = L \cdot I \cdot L' = LL' = \Sigma \tag{B.4}$$

The conclusion comes from the fact that the Cholesky decomposition of a correlation matrix R:

$$R = \begin{bmatrix} 1 & \rho_{SH} & \rho_{SL} \\ \rho_{SH} & 1 & \rho_{HL} \\ \rho_{SL} & \rho_{HL} & 1 \end{bmatrix}$$

can be easily calculated to be equal to Q:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \rho_{SH} & \sqrt{1-\rho_{SH}^2} & 0 \\ \rho_{SL} & \frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}} & \sqrt{1-\rho_{SL}^2-(\frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}})^2} \end{bmatrix}$$

Appendix C. Wage regression

The regressions of log wages by age return the following coefficients.

Table C.7: Wage regression results by age

	flat	moderate	steep
(intercept)	-91.095092	-294.4476	-395.0145
age	9.950874	21.4301	29.9319
age^2	-0.119622	-0.2249	-0.3196

The log wages can be calculated using these coefficients. Figure C.12 illustrates how the estimated rates fit the data:

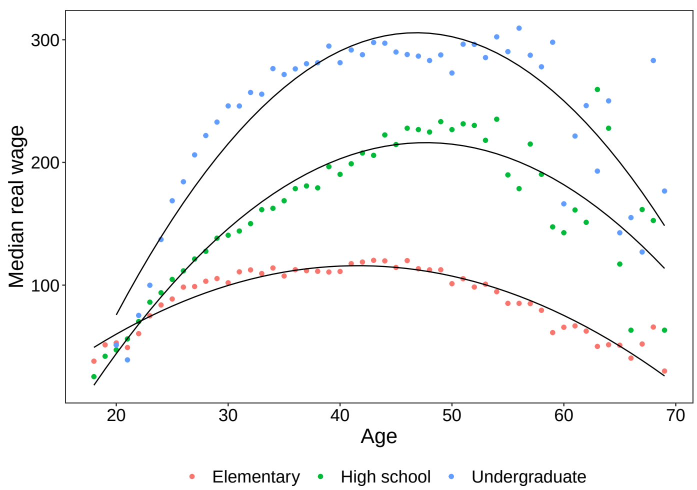

Figure C.12: Fitted values from wage regressions

Appendix D. Portfolio allocations

In this section we present portfolio allocations for individualized life-cycle strategies for heterogeneous individuals, mentioned in the article.

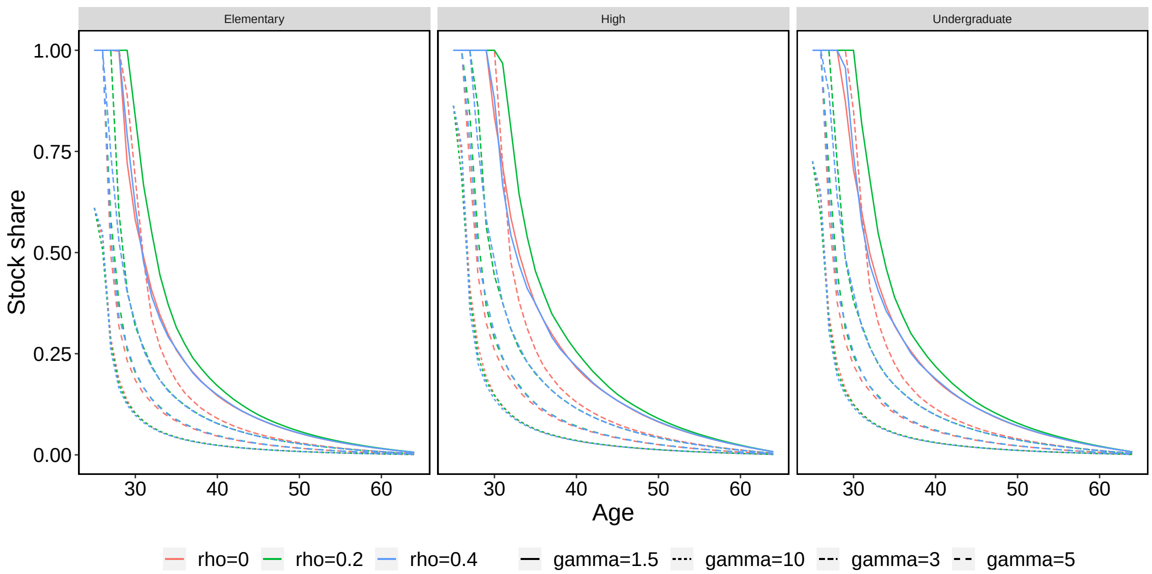

Figure D.13: Stock shares by age for Bodie et al. [4]

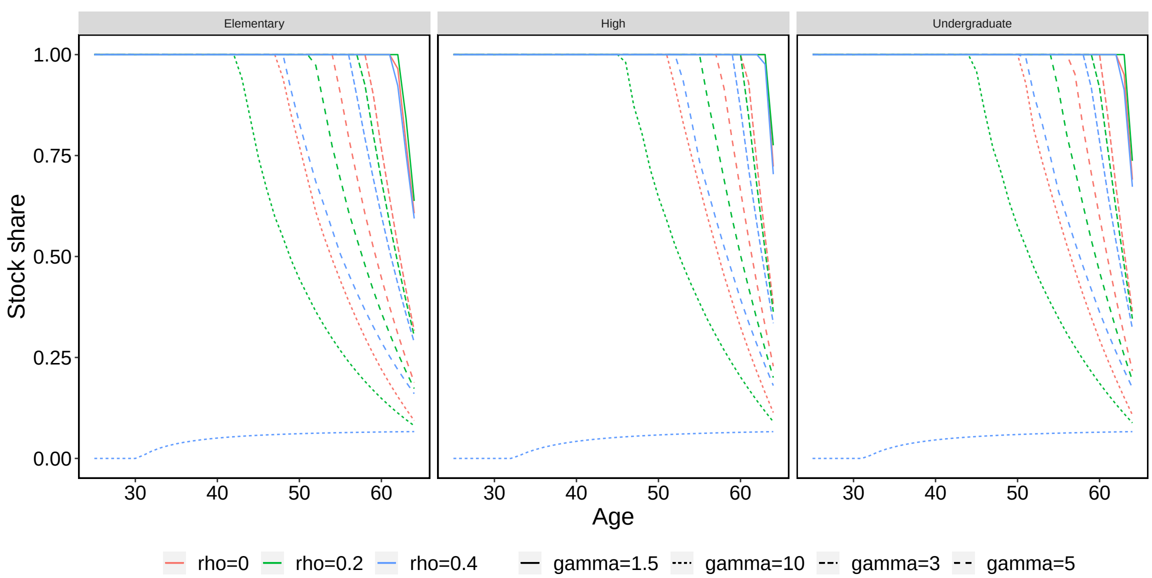

Figure D.14: Stock shares by age for Munk [14] without housing

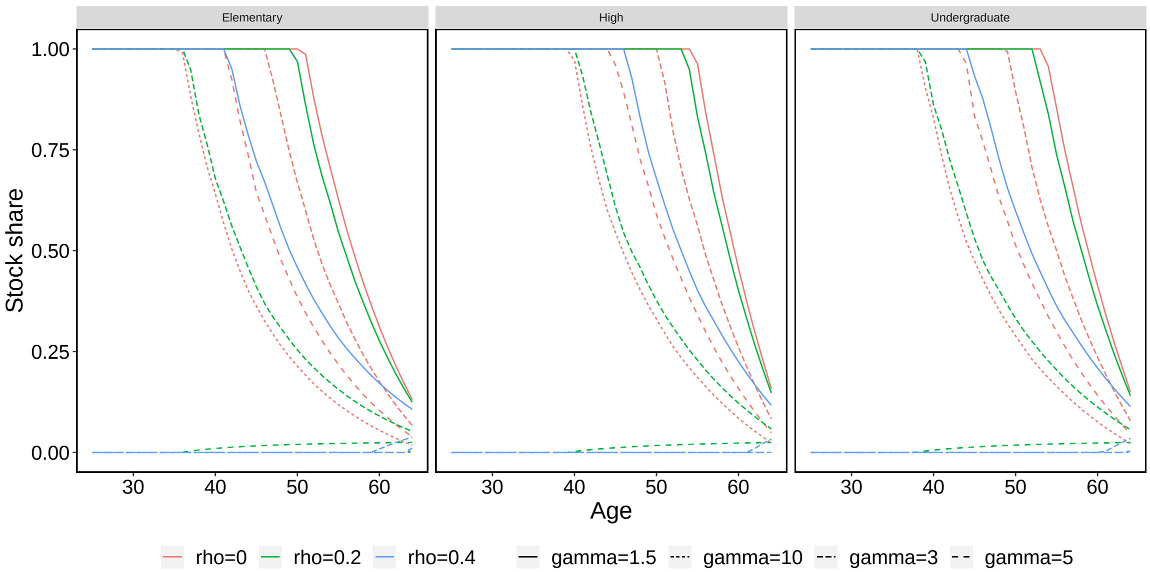

Figure D.15: Stock shares by age for Munk [14] with housing

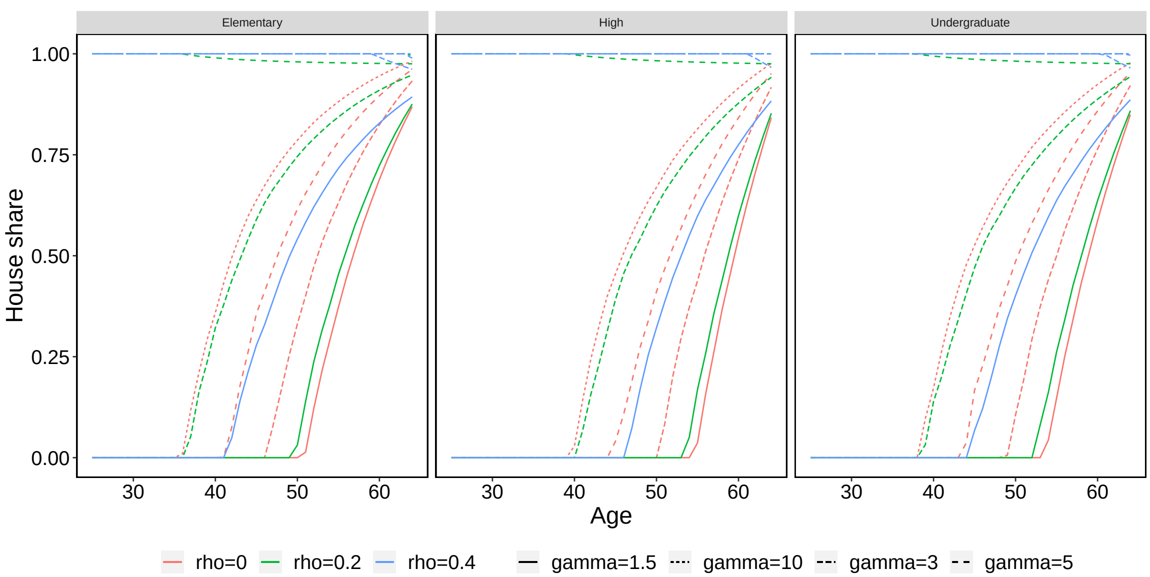

Figure D.16: House shares by age for Munk [14] with housing

Appendix E. Sensitivity analysis

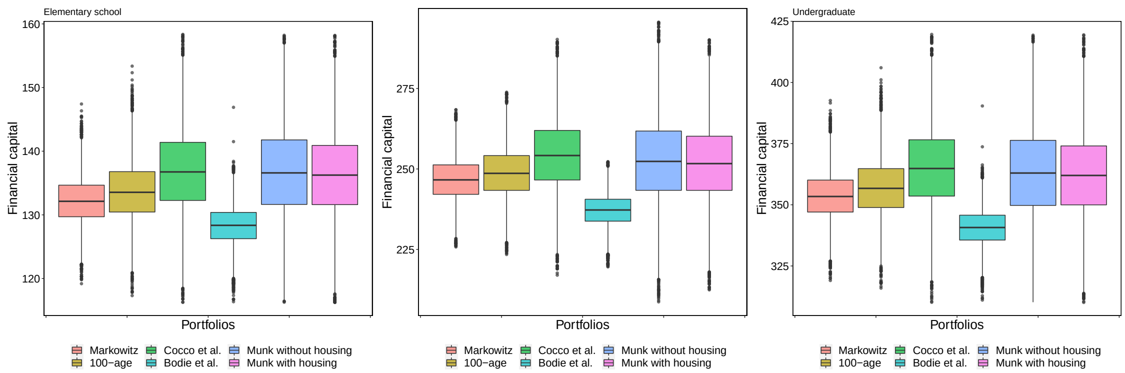

Figure E.17: Accumulated capital W_{65} for 6 investment strategies for flat, moderate, and steep income curves.

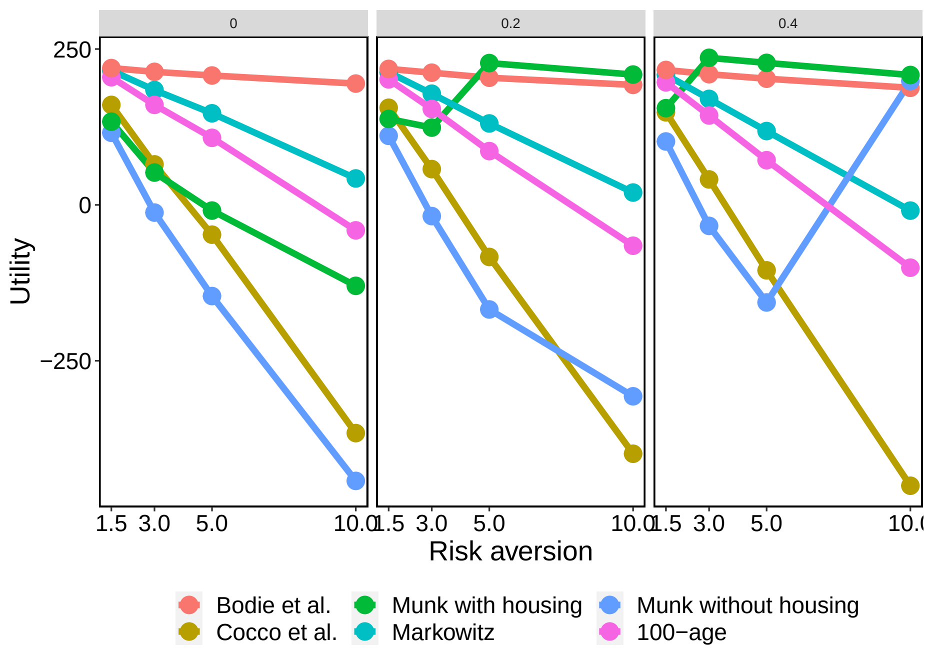

Figure E.18: Sensitivity of utility to the risk aversion for three sectors of work (stock-income correlation).

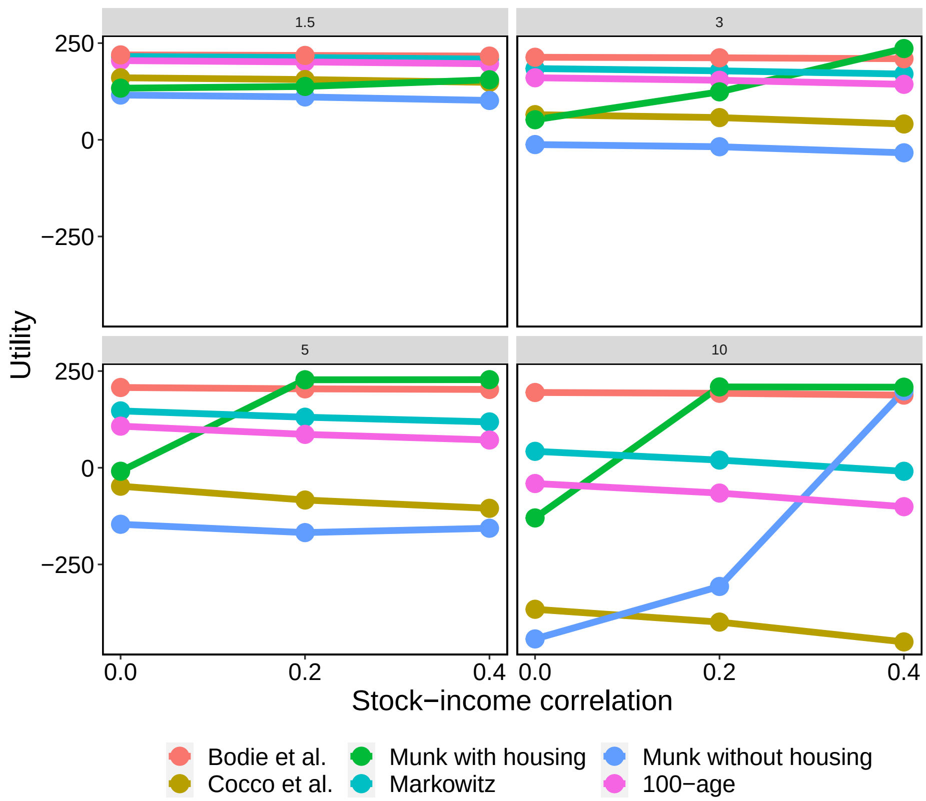

Figure E.19: Sensitivity of utility to the sectors of work (stock-income correlation) for 4 levels of risk aversion.