

# A Demand Based Theory of Income Distribution and Growth\*

Ozan Hatipoglu  
Bogazici University<sup>†</sup>

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## Abstract

This paper builds a demand based theory of inequality and innovation-driven growth in a Schumpeterian setting. When people have hierarchic preferences inequality affects innovation-driven growth through the implied demand distribution over new goods. The paper examines the demand path of the firm through its life-cycle under different growth and patent regimes and analyzes the efficiency of dynamic resource allocation under different inequality scenarios. Unlike previous models, the monopolists are protected by patents of finite length which gives rise to threshold effects in efficient redistributive schemes.

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<sup>†</sup>Department of Economics, Natuk Birkan Hall, Bosphorus University, Bebek, 34342, Istanbul, Turkey. E-mail: ozan.hatipoglu@boun.edu.tr

## 1. Introduction

This paper analyzes the effect of inequality on innovation-driven growth when innovations are protected by patents of finite length. In a static world, inequality affects the market size for the innovators because richer consumers purchase more of the new goods than the poor consumers. In a dynamic setup, changes in inequality bring about demand jumps and demand falls for a particular quality good which affects its innovator's expected profit flow. For instance, a decrease in inequality might push the profit flow earlier in the life cycle, and thereby increasing the net present value of the ongoing innovations.

When there are indivisibilities in consumption, the market size for a product might be affected by a change in inequality or a change in both the level and growth rate of income. However, when patents are finite, incentives of a firm to innovate might not be affected from fluctuations in its market size. In other words, when faced with increased demand, innovators might face also perfect competition when their patents expire early enough. Such a situation might arise either when patent duration is too short or inequality is too high. In this setup there are several new aspects of the relationship between inequality and incentives to innovate that are worth mentioning. First, it is not only the shape of the wealth distribution that is important to growth, but also how long the innovator is expected to survive as a monopolist and thereby will decide to enter. Second, it is not only the level of inequality what matters for the evolution of markets but also how fast it changes. And finally, the growth rate and the duration of monopoly status jointly determine the nature of the relationship between inequality and innovation driven growth.

The effect of income or wealth inequality on incentives to innovate has been analyzed by a small group of researchers who have generally taken the monopoly duration to be infinite (Foellmi and Zweimuller, 2006). This paper extends previous work by relaxing the assumption that innovators are protected by patents of infinite length. In most of the quality-ladder models, this assumption

is made for the purpose of tractability. However, within the set of models where inequality affects innovation-driven growth through demand composition, limited duration of monopolistic power has substantially different implications for the inequality-growth relationship.

For instance, given a low initial level of inequality, an increase in the average income, or a decrease in inequality might cause more inefficiently produced luxuries to be consumed in equilibrium. Resources are thus held up which would be otherwise used in more efficient production. This is an equilibrium with lower growth, which is most likely to occur if the average income is high. In such a situation, reducing inequality beyond a certain threshold reverses the sign of the inequality-growth relationship. On the other hand, if the average income is too low, reducing inequality fosters growth as long as the innovator's product is already consumed by the rich. If the quality good is too expensive for both rich and the poor, then increasing inequality might foster growth by bringing the profit flow earlier so that the innovator can enjoy profits before its patent expires. In this setup, the position of the most recent innovator's entry in the hierarchy of needs and the duration of its status as a monopoly are two important determinants of the relationship between inequality and innovation-growth.

This paper extends the previous studies on the relationship between inequality and innovation-driven growth further by studying the optimal patent length and threshold effects in redistribution. When growth is driven by innovations a redistribution induces higher growth only if the amount of redistribution is above a certain threshold. Making the poor richer induces a higher rate of entry by the innovators only if poor become rich enough to purchase the innovator's good within the good's lifecycle.

This paper is closely linked to Foellmi and Zweimuller (2006) who study the effect of inequality on innovation-based growth when people have non-homothetic preferences. In their setup, once the innovators enter the market, they maintain monopoly positions forever by virtue of infinite patents. The length of the patents in my model, along with inequality and the growth rate, is one of the determinants of the regime in which the economy operates. Therefore patents and the duration of

the monopolistic state determine who can afford an efficiently produced or a luxury product ex-ante.

It is surprising that growth literature has neglected until recently how inequality-determined demand structures affect the incentives to innovate and growth. Murphy, Shleifer and Vishny (1989) stress the role of composition demand as a trigger to the adoption of increasing returns technologies and hence industrialization. Their model differs in that they do not take into account the expansion of the menu of available goods during industrialization. Glass (1996) uses preferences similar to the Grossman and Helpman (1991) model of quality ladders, but assumes that there are two types of households with different tastes about quality. It is because of this assumption that income distribution plays a role. Li (1996) has a set-up which is similar to my model but assumes that consumers differ in their labour endowments which are uniformly distributed across households. The assumption of a uniform distribution has the disadvantage that only one dimension of inequality - the range of the distribution - can be studied. A similar paper is by Chou and Talmain (1996) who consider again a single type demand pattern in a Schumpeterian setup. The demand composition as a general problem has been studied by Baland and Ray(1991) in a static framework. They consider the impact of income distribution on income level, not on growth. In Matsuyama (2002) the different goods are ranked according to priority and new goods are initially luxuries and finally become necessities. In this paper, the model exhibits multiple equilibria as there exists a complementarity between today's and future research. The aggregate growth rate is determined by R&D whereas the incentives to innovate depend on the economy wide growth rate. The effect of demand side effects on innovation incentives have been also studied by Aoki and Yoshikawa (2002) who do not consider the role of income distribution. Greenwood and Mukoyama(2001) analyze how income distribution affects innovation incentives in a partial equilibrium context.

In this paper, growth is driven by innovations which are protected by patents of finite length. Innovations are either new inventions or new goods replacing old products which satisfy the same needs.<sup>1</sup> Innovators are subject to displacement only after their patents expire and they are displaced

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<sup>1</sup>Romer's (87,90) and Grossman and Helpman's (91) models are similar but in their models, innovations take place in the intermediate goods sector and/or innovators introduce new but additional products . Another similar setup can be found in Aghion and Howitt (92) where new products replace old ones but exist only until the next innovation.

by other, more efficient competitive producers if any. The incentives to innovate at any time will depend on the demand structure of the economy through potential market size, the length of patents and the endogenously determined growth rate. A range of the equilibria are characterized by a situation where the poor can not afford all the products available on the market and the innovators face a demand jump but not until the poor are rich enough to purchase the new product. The waiting time, in turn, depends both on the growth rate and the inequality level. If patents expire after the poor can afford the product, then the monopolistic profits are determined by how much after the demand jump the patents will expire. In fact, for how long a monopolist sells its product to the whole population instead of just rich, is not solely determined by the inequality level but also the patent length and the growth rate of the economy.

In the next section I introduce the model. In the third section, I describe the conditions for a general equilibrium. In section four, I present the main arguments of the paper, and in section five policy applications are analyzed. Section six concludes.

## 2 The Model

### 2.1 Technology

There are two types of technology; old and new. The new technology brings increasing returns and is available only after research and development. Old technology, on the other hand, brings constant returns and access to it is free. The new technology is either an innovation which is a new product replacing an existing product or a new method of production for the existing product. The innovator firms are granted patents which allow them to operate monopolistically for some constant amount of time  $\eta$ . In this economy the only factor of production is labour. The old firms require  $a_{old}(t)$  units of labour per output. To make an innovation  $a_{rd}(t)$  units of labour are needed. After innovation the firm can produce with  $a_{new}(t)$  units of labour per output, where  $a_{new}(t) < a_{old}(t)$ . The growth is driven through innovations and  $a_{rd}(t)$  is decreasing in the amount of past innovations due to learning experience. I will also assume that the labour input requirements in both final goods sectors are decreasing in the amount of past innovations a la Young (1993). The input coefficients

at any time  $t$  is then given by

$$a_i(t) = \frac{a_i}{n(t)} \text{ for } i = old, new, rd \quad (1)$$

This specification implies that any increase in the variable  $n(t)$  is a proxy for the increase in the aggregate knowledge stock which spills over uniformly across all sectors<sup>2</sup>. This is assumed to ensure the heterogeneity lies in the demand side which is the focus of my analysis.

## 2.2 Firms

The old firms operate in the competitive sector because of constant returns. At any time, the price of an old firm's product is given by  $p_{old}(t) = w(t)a_{old}(t)$ . where  $w(t)$  is the wage rate determined in a perfectly competitive labour market and is the same for all sectors. For simplicity, I will set the price of competitive products to one for the rest of the article. I'll assume that at the steady state the economy grows at an endogeneous rate of  $g$ . In other words, at the steady state  $n(t)$  is growing at a rate of  $g$  and all labour input coefficients are decreasing at rate  $g$ . Since the productivity growth is uniform across activities of research and production, one can easily show that the wages increase at the same rate and therefore that the marginal production costs for all sectors remain constant over time.

The new firms are granted patents that prevent entry by other firms as long as the patented firm charges the competitive price. The patent allows the firms to operate as monopolists for a certain amount of time. No monopolistic firm has an incentive to charge a price higher than unity which will trigger entry from the competitive fringe. For simplicity, it is assumed that the monopolistic firm does not charge a price lower than unity which translates to monopolist not necessarily choosing the profit maximizing price. The monopolist earns a unit profit of  $\pi = 1 - w(t)a_{new}(t)$ . Using (6) and the wage condition  $\pi = 1 - \frac{a_{old}}{a_{new}} > 0$  which is independent over time. The labour market is competitive and the wage rate is given by  $w(t)$  and is the same for all sectors.

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<sup>2</sup>Note that this differs from Grossman and Helpman (92) where the productivity grows only in research.

### 2.3. Consumers

I assume the consumers have hierarchy over wants  $j \in [0, \infty)$ . A low value of  $j$  is associated with basic needs and a high value of  $j$  is associated with luxurious needs. Let  $c_i(t)$  represent the most luxurious want that consumer  $i$  can afford to and will satisfy. The lifetime utility of the consumer  $i$  can then be written as<sup>3</sup>

$$U_i = \int_0^{\infty} u_i(t)e^{-\rho t} dt = \int_0^{\infty} [1 + \ln(c_i(t))] e^{-\rho t} dt \quad (2)$$

where  $\rho$  represents the discount rate. Consumers own the same wage rate  $w(t)$  but differ in their asset holdings  $W_i(t)$ . I further assume there is a perfect capital market with interest rate  $r$  which is constant at the steady state. The steady state growth rate is  $g$  at which all variables grow including consumption. The consumer's problem can then be written as:

$$\max_{\{c_i\}} U_i \text{ s.t. } \int_0^{\infty} c_i(t)e^{-rt} dt \leq W_i(0) + \int_0^{\infty} w(0)(t)e^{-(r-g)t} dt$$

The optimal consumption path is governed by the following relations:

$$g = r - \rho \quad (3)$$

and

$$c_i(0) = w(0) + \rho W_i(0) \quad (4)$$

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<sup>3</sup>The consumers get  $1/j$  units of additional utility from satisfying want  $j$  when  $j > 1$  and 1 unit when  $j \in [0, 1]$ . Then these preferences can be represented as:  $u_i(t) = \int_0^1 1dj + \int_1^{c_i(t)} \frac{1}{j} dj = 1 + \ln(c_i(t))$

It is obvious from above that the income distribution affects the dynamics and level of an innovator's demand.

Let  $h_p(t)$  ( $h_r(t)$ ) denote the highest ranked want the poor (rich) can afford to satisfy. Then any good  $j \in [0, h_p(t)]$  is bought both by the rich and the poor whereas any good  $j \in (h_p(t), h_m(t)] \cup (h_m(t), h_r(t)]$  is bought only by the rich where  $h_m(t)$  is the highest ranked want satisfied by a monopolist firms product. Clearly, the number of innovations up to date  $t$ ,  $n(t)$ , is equal to  $h_m(t)$ . Moreover, let the range  $[0, h_c(t)]$  denote the firms whose patents have expired and operate now competitively. The range where the monopolistic firms still operate is  $(h_c(t), n(t)]$  and the competitive firms who serve the "fancies of the rich" operate in the range  $(n(t), h_r(t)]$ .

The ranking of the product ranges above relies on the assumption that any want can be satisfied by the products of old firms. The traditional technology is freely available and once there is demand the old (inefficient) producers can enter without any costs. The new producers on the other hand aim at markets which have the highest growth potential to incur the R&D costs. This implies that the "fancies of the rich" market is served by the competitive producers whereas the middle and the higher middle class is served partly by the monopolistic producers. The "poorest" market is again served by competitive producers with expired patents. This is shown in Figure 1. Everything grows at a constant rate and the entry occurs at time  $t$  at point  $n(t)$  and at time  $t + \Delta t$  at  $n(t + \Delta t)$ . At any time, there are three types of demand structure that will determine size of these markets.

Case I:  $h_p < h_c < n$  Only the rich can afford monopolistic firms products; in other words the firms patent expires before the poor can afford its product

Case II:  $h_c < h_p < n$  At least some part of the monopolistic sector products can be afforded by the poor.

Case III :  $h_c < n < h_p$  Both rich and the poor can afford all monopolistic firms products.

Note that this specification also represent initial inequality levels in affordability or consumption.

Figure 1 Here

## 2.4 Income Distribution and Demand Structure:

In this section I closely follow Zweimuller and Foellmi (2006) who consider a simple distribution of wealth with two groups; rich ( $r$ ) and poor ( $p$ ). Both rich and poor earn the same wage but own different wealth levels. Let  $\gamma_i$  be the ratio of consumer  $i$ 's wealth to the average wealth,  $\gamma_i = \frac{W_i}{\bar{W}}$ ,  $i = p, r$ . If the population share of the poor is  $\phi$  than the fraction of aggregate wealth hold by the poor is  $\phi\gamma_p$  whereas the rich hold a fraction of  $(1 - \phi)\gamma_r$ . Since  $\phi\gamma_p + (1 - \phi)\gamma_r = 1$ , one can write

$$\gamma_r = \frac{1 - \phi\gamma_p}{1 - \phi} \quad (5)$$

This relation represents the Lorentz curve, Note that an increase in  $\gamma_p$  and a decrease in  $\phi$  (holding  $\gamma_p$  constant) leads to less inequality.

In this economy, aggregate wealth refers to holdings of firm shares. Let  $v_k(t)$  be the present value of firm  $k$ 's profit. The aggregate value of the wealth at any time  $t$  is then given by  $V(t) = \int_0^{n(t)} v_k(t) dk$ . The value of asset holdings of each consumer is given by

$$W_i(t) = \frac{\gamma_i V(t)}{L} \quad i = p, r \quad (6)$$

where  $L$  is population. Before moving on to entry and exit decision and the value determination of the firm one needs to analyze the demand structure which will determine the output levels in both sectors. More specifically, one can ask which needs are satisfied by which sector. At any time, the market size for each firm will depend on the number of people who can afford its good. The incomes grow over time and some firms anticipate a larger market because more people will be able to afford their good within near future. Therefore the market size and expected profits for each firm will be determined by the overall growth rate of the economy, the patent length and the level of inequality.

The entry decision is then made only if the expected profits are large enough to incur the R&D costs.

## 2.5. The Resource Constraint

Note that the range  $(h_c(t), n(t)]$  will be determined by the growth rate,  $g$ , and the patent length,  $\eta$ , in the following manner;  $n(t + \eta) = n(t)e^{-g\eta} = h_c(t)$ . It follows that  $\eta = -(1/g) \ln(h_c(t)/n(t))$ . Note that the longer the patent length the smaller the ratio  $h_c(t)/n(t)$ . Moreover, in the steady state this ratio is solely determined by the patent length since the growth rate becomes a constant.

Let  $Y_i (i = new, old)$  denote the total production in each sector. Since the labour is the only factor of production the employment in the R&D, the new and old sectors are  $L_{R\&D} = \dot{n}(t)a_{rd}(t)$ ,  $L_{new} = Y_{new}(t)a_{new}(t)$  and  $L_{old} = Y_{old}(t)a_{old}(t)$  respectively. The total employment is then

$$L = \dot{n}(t)a_{rd}(t) + Y_{new}(t)a_{new}(t) + Y_{old}(t)a_{old}(t) \quad (7)$$

In a steady state situation  $h_p, h_r$  and  $n$  grow at the same rate. Using the definitions  $d_p = \frac{h_p(t)}{n}$ ,  $d_r = \frac{h_r(t)}{n}$ ,  $d_c = \frac{h_c(t)}{n}$ ,  $g = \frac{\dot{n}}{n}$  and equation (7). We can write the resource constraint in each type of demand structure above respectively as<sup>4</sup>:

$$\text{Case I : } L = a_{rd}g + a_{new} [(1 - \phi)(1 - d_c)] L + a_{old} [(1 - \phi)(d_r - 1)L + d_p] \quad (8)$$

$$\text{Case II : } L = a_{rd}g + a_{new} [\phi d_p + (1 - \phi)(1 - d_c)] L + a_{old} [(1 - \phi)(d_r - 1)L + d_c]$$

$$\text{Case III : } L = a_{rd}g + a_{new}(1 - d_c)L + a_{old} [(1 - \phi)(d_r - 1)L + \phi L(d_p - 1) + d_c]$$

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<sup>4</sup>Note that for each case the output can be written as follows:

Case 1:  $Y_{new}(t) = (1 - \phi)L(n(t) - c_c(t))$ ,  $Y_{old}(t) = (1 - \phi)L(c_r(t) - n(t)) + c_p(t)$ . Case 2:  $Y_{new}(t) = \phi L c_p(t) + (1 - \phi)L(n(t) - c_c(t))$ ,  $Y_{old}(t) = (1 - \phi)L(c_r(t) - n(t)) + c_c(t)$ . Case 3:  $Y_{new}(t) = L(n(t) - c_c(t))$ ,  $Y_{old}(t) = (1 - \phi)L(c_r(t) - n(t)) + \phi L(c_p(t) - n(t)) + c_c(t)$

## 2.6. The Value of A Monopolistic Firm

With ongoing innovations and growing incomes a monopolist  $j$  decides whether or not to enter the market. Let  $\eta$  denote the length of the patent the monopolist is granted in case it enters the market. Let  $\mu$  denote the length of time it takes until the poor can afford to buy good  $j$  which is consumed only by the rich at the time of decision. The value of the most recent innovator  $j$  can then be written as

$$v_j(t) = \pi \int_t^{t+\eta} D_j(\tau) e^{-r(\tau-t)} d\tau$$

$$= \begin{cases} \pi \int_t^{t+\eta} (1-\phi) L e^{-r(\tau-t)} d\tau & \text{for } \mu > \eta \\ \pi \int_t^{t+\mu} (1-\phi) L e^{-r(\tau-t)} d\tau + \pi e^{-r\mu} \int_{t+\mu}^{t+\eta} L e^{-r(\tau-(t+\mu))} d\tau & \text{for } 0 < \mu \leq \eta \\ \pi \int_t^{t+\eta} L e^{-r(\tau-t)} d\tau & \text{for } \mu \leq 0 \end{cases}$$

where  $D_j(\tau)$  is demand for firm  $j$  at time  $\tau$  and  $L$  stands for population. Three possible values refer to the cases i) The monopolist serves only the rich until patent expires ii) The monopolist serves first the rich and then the whole population until the patent expires iii) The monopolist serves the whole population until the patent expires. Note that as long as the patents do not expire before the poor can afford the firms product there will be a jump in the expected demand for the product at date  $t + \mu$  from  $(1 - \phi)L$  to  $L$ . Solving the above integral yields:

$$v_j(t) = \begin{cases} \frac{\pi L}{r} (1-\phi)(1-e^{-r\eta}) & \text{if } \mu > \eta, \text{ or} \\ \frac{\pi L}{r} [(1-\phi) + \phi e^{-r\mu} - e^{-r\eta}] & \text{if } 0 < \mu \leq \eta \\ \frac{\pi L}{r} (1-e^{-r\eta}) & \text{if } \mu \leq 0 \end{cases} \quad (9)$$

Given  $\mu \leq \eta$ , it is clear that a shorter  $\mu$  implies less discounting and a higher value for the firm.

The value of the firm when the poor is able to afford the product becomes  $\frac{\pi L}{r} [1 - e^{-r/\eta-\mu}]$ . This

value will then drop until the patent expires. The amount of time,  $\mu$ , that will pass until the jump in demand can be found by making use of the following relation;  $h_p(t + \mu) = h_p(t)e^{g\mu} = j$ .

$$\mu = -(1/g) \ln(h_p(t)/j) > 0 \quad (10)$$

A higher growth rate or a smaller distance between the most advanced good that the poor can afford and the monopolist firm's product  $j$  imply a shorter time until the demand jump. Substituting  $d_p(j) = \frac{h_p(t)}{j}$  and equation (0) in (9) gives: :

$$v_j(t) = \begin{cases} \frac{\pi L}{r}(1 - \phi)(1 - e^{-r\eta}) & \text{if } \mu > \eta \quad , \text{ or} \\ \frac{\pi L}{r} \left[ (1 - \phi) + \phi d_p^{\frac{r}{g}}(j) - e^{-r\eta} \right] & \text{if } 0 < \mu \leq \eta \\ \frac{\pi L}{r}(1 - e^{-r\eta}) & \text{if } \mu \leq 0 \end{cases} \quad (11)$$

Regardless of the inequality level a higher growth rate has two effects: 1) A higher interest rate. The future profits have to be discounted at a higher rate which reduces the firm's value. 2) A shorter time period until the poor can afford the firm's product. The sales to the poor are realized faster which increases the profitability of innovation, hence the firm's value. However, if the inequality is sufficiently high then an increase in demand will never occur because firm's patent will expire before the poor can afford to buy the product. In this case the second effect diminishes. Thus a higher growth rate means a lower value for the monopolist when it enters the market. If the inequality, however, is sufficiently low, which makes it possible for the poor to buy the product already at the time of entry, the second effect again diminishes. In this case, instead of the rich only the product of the firm is bought by the whole population until its patent expires. There is a range of growth rates where the second effect dominates the first one as shown in Figure 2.

Figure 2 Here

Figure 3 Here

## 2.7. Entry, Exit and the Partial Equilibrium

Let's consider the most recent innovator  $n$ . For a profitable entry the innovation costs ( $a_{rd}(t)w(t) = \frac{a_{rd}}{a_{old}}$ ) should not exceed the reward to an innovation. Assuming free access to R&D technology implies there are zero profits in equilibrium. That is  $\frac{a_{rd}}{a_{old}} \geq v_n(t)$ . When the innovations take place this condition holds with equality. This zero profit condition implies that the current costs of an innovation must not be smaller than current returns from an innovation which are simply the innovation costs discounted by the interested rate. Moreover, in equilibrium the interest rate is determined by the growth rate as in equation (2). The entry condition can be written as:

$$\frac{a_{rd}}{a_{old}}(g + \rho) = \begin{cases} \pi L(1 - \phi)(1 - e^{-(g+\rho)\eta}) & \text{if } \mu > \eta \quad , \text{ or} \\ \pi L \left[ (1 - \phi) + \phi d_p^{\frac{g+\rho}{g}} - e^{-(g+\rho)\eta} \right] & \text{if } 0 < \mu \leq \eta \\ \pi L(1 - e^{-(g+\rho)\eta}) & \text{if } \mu \leq 0 \end{cases} \quad (12)$$

where  $d_p = d_p(n) = \frac{h_p(t)}{n}$  is the poor's consumption position. The possible equilibria are shown in Figure 3. Let  $R$  denote the right side and  $C$  the left side of equation 12.  $R$  represents the current returns from an innovation  $C$  represents the current costs. The growth rate is shown on the x-axis and current costs and returns are shown on the y axis. The high initial inequality line refers to the case  $\mu > \eta$ , the upper and lower bounds for the returns is shown by I. The low initial inequality line refers to the case when  $\mu \leq 0$  (III) and the medium initial inequality line refers to the case when  $0 < \mu \leq \eta$  (II). Note that, the second case represents the partial equilibrium in Zweimuller(2000) as a special case.

The C curve represents the current costs of innovation as above and is formulated by the left hand of the equation. The R curve represents current returns from an innovation as above.  $R_I, R_{II}$  and  $R_{III}$  are drawn with the same time preference, unit profit and population. The curve above  $R_{II}$  is drawn with a higher  $d_p$  ratio. The C curve intersects with the y axis at  $\rho \frac{a_{rd}}{a_{old}}$  and has a slope of  $\frac{a_{rd}}{a_{old}}$ . The shape and curvature of the R curve depends on the time preference and the initial demand

structure the firm is facing. The intercept of the R curve is  $\pi L(1 - \phi - e^{-\rho\eta} + \phi e^{-\rho\eta})$  for  $\mu > \eta$  (I),  $\pi L(1 - \phi - e^{-\rho\eta})$  for  $0 < \mu \leq \eta$  (II),  $\pi L(1 - e^{-\rho\eta})$  for  $\mu \leq 0$  (III).

Figure 4 Here

Figure 5 Here

Note that the upper and lower bounds for the firms returns (I) when there is high initial inequality are contained in II. As  $g$  goes to zero the returns are lower when there is less inequality. The analysis of Figure 3 indicates that there is a possibility of multiple equilibria depending on the time preference at all levels of inequality. Starting with a medium level of inequality (II), a higher inequality situation (I) (an increase in inequality) might lead to a higher entry rate ( $g_2$ ) than the one at a lower inequality situation (II) ( $g_1$ ) when the time preference is high enough. It might lead to a lower entry rate if the time preference is sufficiently small. ( $g_3 \rightarrow g_2$ ) Similarly starting with the low inequality (III) an increase in inequality might lead to a higher entry rate if the time preference is sufficiently high and vice versa.

### 3. General Equilibrium

#### 3.1. Characterization

In this section the general equilibrium is characterized and wealth distribution is incorporated into the analysis. The case of a unique equilibrium is analyzed and finally, multiple equilibria and its implications regarding to the main arguments of this paper are considered. A general equilibrium in this model is a situation in which the following conditions hold simultaneously.

- i) Consumers maximize their life-time utility subject to their temporal budget constraint and initial wealth.
- ii) Firms maximize profits.

iii) The resource constraint holds.

The firm's and the consumers optimal decisions and the zero profit equilibria were dealt in previous sections. Equation (12) states the partial equilibrium condition on the firms' side. The optimal consumption choice in (3) and the resource constraint in (8) together with the partial equilibrium condition in (12) form a set of equations which describe the general equilibrium in this economy. To incorporate the wealth distribution into the general equilibrium framework I also make use of equation (5)<sup>5</sup>. Using algebra, which is omitted here, the system can be reduced to following equations in the unknowns of  $d_p$  and  $g$  for each case.

$$d_p = \begin{cases} \frac{L}{a_{old}} \left[ \frac{\gamma_p [1-(1-\phi)[a_{new}(1-d_c)-(a_{old}-1)] + \gamma_r(1-\phi)}{(\gamma_r(1-\phi)L + \gamma_p)} \right] - g \frac{a_{rd}\gamma_p}{a_{old}(\gamma_r(1-\phi)L + \gamma_p)} & \text{if } \mu > \eta \\ \frac{1}{a_{old}} + \gamma_p \left[ \frac{L - a_{old}d_c - (1-\phi)L[a_{new}(1-d_c) + 1 - a_{old}] - \frac{a_{new}L\phi}{a_{old}}}{L[a_{old}(1-\phi)\gamma_r + a_{new}\phi\gamma_p]} \right] - g \frac{a_{rd}\gamma_p}{L[a_{old}(1-\phi)\gamma_r + a_{new}\phi\gamma_p]} & \text{if } 0 < \mu \leq \eta \\ \frac{1}{a_{old}} + \gamma_p \left[ \frac{L - a_{new}[L(1-d_c)] - a_{old}d_c - (1-a_{old})L}{a_{old}L} \right] - g \frac{a_{rd}\gamma_p}{a_{old}L((1-\phi)\gamma_r + \phi\gamma_p)} & \text{if } \mu \leq 0 \end{cases} \quad (13)$$

where  $(1-\phi)\gamma_r + \phi\gamma_p = 1$ .

$$\frac{a_{rd}}{a_{old}}(g + \rho) = \begin{cases} \pi L(1-\phi)(1 - d_c^{\frac{g+\rho}{g}}) & \text{if } \mu > \eta, \text{ or} \\ \pi L \left[ (1-\phi) + \phi d_p^{\frac{g+\rho}{g}} - d_c^{\frac{g+\rho}{g}} \right] & \text{if } 0 < \mu \leq \eta \\ \pi L(1 - d_c^{\frac{g+\rho}{g}}) & \text{if } \mu \leq 0 \end{cases} \quad (14)$$

Equation (13) is simply a negative linear relation between  $g$  and  $d_p$  which holds in equilibrium. It reflects the optimal consumption choices and the resource constraints. The equilibrium condition in (14) is a restatement of no-profit condition as in (12). Note that in (14) for  $\mu > \eta$  and  $\mu \leq 0$ , the growth rate is independent of the poors's position  $d_p$  which is as expected. Below I present the

<sup>5</sup>Dividing both sides of equation (10) by  $n$  and substituting equation (3) and equation (2) one gets

$$\frac{c_i(0)}{n} = \frac{w_0}{n} + \rho \frac{A_i(0)}{n} = \frac{1}{a_{old}} + \rho \frac{\gamma_i V(0)}{nL} \text{ or}$$

$$d_i = \frac{1}{a_{old}} + \rho \frac{\gamma_i \bar{v}}{L} \quad (2')$$

where  $\bar{v} = \frac{V}{n}$  and is constant. I solve for  $\bar{v}$  after substituting for  $d_i$  ( $i = p, r$ ) in resource constraints for each case (5). Substituting  $\bar{v}$  back to (2') and rearranging gives (15)

figures that represent the above conditions.

Figure 4 depicts the possible equilibria of the model. The P curves represent equation (14) and Q curves represent equation (13). The numbers I, II and III refer to the initial demand patterns (cases) when  $\mu > \eta$ ,  $0 < \mu \leq \eta$  and  $\mu \leq 0$ , respectively. First, let me discuss the shapes of P and Q curves. The first case represents the situation, in which, whatever the patent length, the poor can not afford to buy the monopolists product during its lifecycle. The patents expire before the poor is able to afford the product. The poor's position,  $d_p$ , is irrelevant to the firm's decision. In other words, any change in growth rate which will change the poor's position will in turn not affect the firm's prospects in that it doesn't expect a demand jump in the future. This is shown by  $P_I$ .

The definition of  $d_p$  also requires that above a certain level ( $d_p = 1$ ) the entering firm's products will be bought by the poor regardless of the growth rate or the patent length during its lifecycle. This case is shown by  $P_{III}$ . For given levels of patent length and poor's population share equation (14) implies that  $P_{III}$  lies to the right of  $P_I$ .

If there is a possibility of a demand jump during the lifecycle of the firm, then the poor's position will determine the entry rate. The poor's position and the growth rate will be linked in a non-linear way as in the second line of equation(14). This situation is shown by  $P_{II}$ .

For all levels of  $d_p$  the Q curve is a negatively sloped, straight line. The reason that a higher  $d_p$  implies a lower growth rate in each case is due to more resources being diverted towards production of consumer goods instead of R&D. Using equation (13) it is possible to show that the  $d_p$ - intercept of Q becomes larger when moving from I to III. The intuition is simply that when there is no growth all resources are used for consumption which solely determine the poor's position  $d_p$ .

For given parameters, the slope of Q becomes larger when switching from I to II, but becomes smaller again when switching from II to III with III still being higher than I. This deserves some discussion. The slope of Q is determined by the fraction of resources diverted to production of consumption goods and, except case III, by the inequality level. If for a unit forgone consumption

more growth can be achieved than it must be the case that initially few resources are diverted to R&D . The slope of Q is small, as in case I. Note that the fraction of resources diverted to production of consumption goods is higher in case I and III than in II to achieve the same level of increase in  $d_p$ . The intuition here is twofold: First, for both the poor and the rich it is optimal to smooth consumption whenever there arises a possibility of an increase in wealth and consumption in the near future by doing so. In such a case the slope of the Q curve will be higher. Second, diverted resources are increasingly used in the new goods sector, which is more efficient. This reduces the R&D resources which have to be sacrificed in order to achieve the same level of consumption, In case III the first effect is absent, and in case I both effects are absent. As a result, the slope of  $Q_{II}$  is higher than  $Q_{III}$  both of which are higher than  $Q_I$ .

The equilibrium  $(d_p, g)$  pair is found at the intersection of P and Q curves.

### 3.2. Uniqueness

I will discuss the uniqueness of the equilibrium briefly before moving to the main arguments of this paper. A unique general equilibrium exists if and only if the conditions on time preference are such that  $d_p$  is continuous and strictly increasing in growth rate in the no-profit condition (14). Such a situation is presented by the thick line in Figure 4. As the growth rate changes, it is possible to switch from one case to another. Uniqueness of equilibrium requires that the switch, which is shown by the arrows, to be continuous without any jumps in growth rate. In other words, a  $(d_p, g)$  pair should exist above which the  $P_I(P_{II})$  curve smoothly connects to  $P_{II}(P_{III})$ .

In case I and case III the equilibrium growth rate is solely determined by the positions of  $P_I$  and  $P_{III}$ . This is evident in equation 16 where the first and the third line does not include the term  $d_p$ .

Each equilibrium situation depends on the length of the time period  $\mu$  which is determined not only by the growth rate but also the poor's purchasing power and the patent length  $\eta$ .

### 3.3. Multiple equilibria

If we relax the conditions on the time preference the model exhibits multiple equilibria as shown in Figure 5. An increase in the time preference,  $\rho$ , changes the intercept and the curvature of  $P_{II}$  curve whereas it shifts  $P_I$  and  $P_{III}$  to the left as shown by the arrows. Multiple equilibria occurs only in case II when for a sufficiently large  $\rho$  there exist more than one zero-profit equilibria.

#### 4. Main Arguments

In this section the model's predictions on the links between inequality and growth are analyzed. Further consideration is given to the effects of patent length and redistribution policies on growth at different levels of inequality. Since an analytical solution is not attainable, the arguments are provided based on the numerical methods and associated graphs. I distinguished earlier between the three initial conditions (I,II,III) and their implications to the model's behavior. Below, the effects of inequality on growth in each of those cases are analyzed. Case I and case III deals with cases of very high and very low inequality whereas case II deals with a wide range of inequalities from high to low.

Since, the patent length determines the state of this economy it is initially kept fixed to isolate the effects of inequality on growth. Later, this assumption is relaxed to consider the effects of patent length on growth at each level of inequality. Both the poor's share of wealth  $\gamma_p$  and the poor's share of population,  $\phi$ , are used as a proxy to inequality<sup>6</sup>. One has to distinguish between the two different proxies because the effect of inequality may become ambiguous when the poor purchase before the patents expire. A higher group share of the rich would increase markets for new innovators but at the same time would shorten the time during which the innovator has the full market.

In a real world situation, one should keep in mind that case I and case III are rather "extreme" cases. Consider case III, for instance: the assumption that everybody can afford every new firm's product in every equal society will be hard to justify. Nevertheless, for an egalitarian country with high purchasing power and long patents (e.g. Sweden) inclusion of such analysis seems suitable.

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<sup>6</sup>Note that whether looking effects of  $z_p$  or  $\phi$  the following Lorentz relation has to hold:  $\phi z_p + (1 - \phi)z_r = 1$ . Any increase in  $z_p$  is coupled by an appropriate decrease in  $z_r$  keeping  $\phi$  fixed.

Similarly, the idea that new ‘innovations’ are not consumed by the poor before the expiration of patents is also objectionable. However, examples of countries exist where the purchasing power is low enough that some new products are not consumed at all. Between those extremes the model exhibits a non-linear relation between inequality and growth for a wide range of inequalities. The following propositions follow.

**Proposition 1:** *When inequality is very high, a lower population share of the poor( $\phi$ ) leads to a higher growth rate. An increase in the wealth of the poor ( $\gamma_p$ ) leads to a higher growth rate only if the increase is above a certain threshold.*

Proof. See appendix.

**Proposition 2:** *When inequality is high, a lower population share of the poor( $\phi$ ) leads to an unambiguous increase in growth rate, whereas an increase in the wealth of the poor( $\gamma_p$ ) leads to an increase in the growth rate.*

Proof: See Appendix.

**Proposition 3:** *When inequality is low, a redistribution lower population share of the poor( $\phi$ ) leads to an unambiguous reduction in the growth rate. An increase in the wealth of the poor( $\gamma_p$ ) reduces growth rate only if the initial consumption position of the poor relative to the entry position of the monopolist firm is sufficiently small.*

Proof. See appendix.

**Proposition 4:** *When inequality is very low, a lower population share of the poor( $\phi$ ) does not have any effect on the growth rate. The effect of an increase in the wealth of the poor( $\gamma_p$ ) on the growth rate is ambiguous.*

Proof. See appendix.

## 5. Policy Applications

### 5.1. Patent Length and Growth

The effects of patent policies on growth has attracted a lot of discussion in the literature. On the one hand patents create mark-ups distorting relative prices thus reducing welfare, on the other hand they stimulate R&D by increasing profitability of innovations and cause dynamic efficiency gains. For tractability purposes, in this model it is assumed that the firms do not mark-up during their finite monopolistic life which implies that the optimal patent length is infinite, a result already confirmed by Grossman and Helpman (1991). By imposing finite patent lengths, however, one is able to distinguish between different levels of inequality which will matter to the firm's decision to innovate.

An increase in patent length unambiguously increases the growth rate and the poor's consumption. To exemplify the situation case II is shown in Figure 13. Longer patents increase the profitability of an innovation and reduce the effect of inequality by increasing the likelihood that the firms benefit from a future demand jump from the sales to the poor and by allowing them to operate monopolistically over a larger time span. P curve shifts right. More resources are diverted to R&D and growth increases. The increased wealth of the poor reflects in its consumption decision

One interesting issue is how the patent length and growth related at different levels of wealth inequality. Figure 14 shows the patent length as proxied inversely by  $d_c$  on y-axis and growth on x-axis. The figure is drawn after the model is adjusted to the feasible inequality parameters for Case II. There is a nonlinear positive relation between patent length and growth. The growth is maximized at  $d_c = 0$  which amounts to infinite patent length. As the wealth inequality increases the curve shifts outward and its slope becomes flatter This implies that at high levels of inequality a unit increase in patent length increases growth more than it does at low levels of inequality. When the poors' population share is high the potential increase in profitability in case of a demand jump is higher. An increase in patent length makes such higher gains more likely.

Another interesting issue is the role of time preference on patent length - growth relationship. In Figure 15 an increase in time preference shifts the curvature towards x-axis. When people's time preference is low increasing patent length is more effective. When patent lengths increase so does the wealth in an economy. With low preferences the innovators market sustain for a longer time, thus growth is higher.

Figure 6 Here

Figure 7 Here

Figure 8 Here

Figure 9 Here

## 5.2. Redistribution and Income

The political economy models of inequality and growth postulate that in democracies higher inequality leads to higher redistribution through the voting mechanism and higher redistribution hurts growth.<sup>7</sup> In this model, a redistribution from the rich to the poor increases growth as long as it enlarges the innovators potential market. Redistribution will cause exits from the high end competitive fringe reducing static welfare but as a trade-off it induces entry to the R&D sector increasing growth. Overall, a redistribution increases growth except in third case where the poor's purchasing power is already high and a further redistribution causes the resources to be diverted to the production of less efficient goods.

Limited monopoly duration introduces threshold effects in redistributive policies. In a dynamic context, the size of the markets are determined by the patent lengths and with the hierachic preferences not every redistribution is high enough to make the poor purchase the efficient products during the products' lifetime. For instance, at a high inequality situation ,a sufficient redistribution

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<sup>7</sup>There are several versions of this argument in Alesina and Rodrik(1994), Persson and Tabellini (1994), Benabou(1996), Perotti (1996) and Benhabib and Rustichini(1996).

to increase growth is the one which increases the wealth of the poor by at least 232% when the patent length parameter ,  $d_c$  , is equal to 0.2.<sup>8</sup>

Finally, as incomes grow the relationship between inequality and growth reverses sign and becomes positive. The exercise above suggest that this relationship is independent of the time preference parameter as long as the position of the entry by the monopolist falls at or below the poor's highest affordable good. When people are rich enough to consume all monopolists product increasing poor's share of wealth decreases growth by a reallocation of resources towards more inefficient production. This is shown in Figure 9. This result is consistent with the observation by Barro(2000) that inequality and growth are positively related in rich countries and negatively related in poor countries.

## 6. Conclusion

The inequality in purchasing power has received little attention in the literature with respect to its relation to innovation driven growth. The fact that rich economies can support large markets for new products despite large differences of wealth within their populations might cloud any evidence on the link between inequality and growth. This paper considers both aspects of inequality within one framework. It also allows to isolate the effects of one type of inequality from the other.

I presented above a general equilibrium model which accounted for the nonlinear relationship between inequality and growth as evidenced by recent empirical literature. The demand for new products and wealth distribution is closely linked when people have hierarchic preferences. At high and low levels of inequality, an increase in inequality has opposite effects on growth. The mechanics of the model is such that the entering firm's potential market size determines the level of growth, whereas the market size is determined by the inequality level, the finite patent length and the growth rate. Furthermore it is shown that in this setup an increase in patent length increases growth. The magnitude of this effect differs with existing inequality level and time preference. The optimal patent length turns out to be infinite because monopolist firms do not mark-up during their monopolist life.

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<sup>8</sup>When the growth rate is 5% this is equivalent of a patent length of 32 years. With a 3% growth rate it is equivalent to 53 years.

This is assumed for tractability purposes. Even though the inequality is higher in some rich countries compared to poor ones, the purchasing power of the poor in the rich country is high enough to support new innovations. This might explain the empirical discrepancies found in the literature. I also find that the inequality and growth relationship depends on the level of income. As people get richer, the demand patterns may lead to an inefficient allocation of resources and decreasing inequality at high levels of income results in lower growth.

The model presented here is open to modifications and further enhancements. For instance, allowing for mark-ups creates a possibility for a finite optimal patent length due to static price distortions. The negative welfare effect of distorted relative prices would then counterbalance the dynamic efficiency gain of infinite patents. A general solution of the model will considerably be more difficult, however. First the price structure would not reflect the cost structure and consumer purchases would not be efficiently spread over varieties. The equilibrium could then be described by non-linear delayed differential equations.

## **Appendix**

### *Proof of Proposition 1*

Keeping the poor's consumption position fixed, a lower population share of the poor unambiguously increases the growth rate as in Figure A1. The demand is higher for the entering firms because more people can afford their products. This increases profitability and induces more entry. P curve shifts right ( $P_I \rightarrow P'_I$ ). At the same time the demand for the consumption goods by the poor falls but the demand for all consumption goods increase. This is simply due the change in the composition of population. The wealth position of the rich is now worse due to Lorentz relation. The composition of demand shifts towards new and more efficient goods instead of 'luxuries' leaving more resources for R&D. For a unit forgone consumption by the poor, more growth can be achieved because of this increased efficiency. This means that the new Q curve ( $Q'_I$ ) has a lower slope. At the new equilibrium, the growth rate ( $g'_1$ ) is higher. The effect on poors consumption depends on the efficiency of the new

technology. The more efficient the new technology the higher is the poor's consumption.

When the inequality is very high, a small increase in the wealth of the poor does not affect the growth rate. A small increase means that the wealth increase is insufficient to make the poor's consumption choices differ enough to include the new goods before their patent expiration. As long as the poor's consumption stays within a certain range, the economy will operate as in case I and any effects on profitability are ruled out. As a result  $P_I$  curve does not shift. The range of wealth for which an increase does not effect growth can be found by making use the of the equilibrium conditions and the Lorentz equation (2). A sufficient increase in wealth which would switch the model from I to II and increase the growth rate must be at least 232% using the parameters specified as above for  $d_p \leq d_c = 0.2$ ,

In Figure A2 a small increase in wealth rotates the curve  $Q_I$  clockwise around  $(g_1, d_{p1})$  but does not affect the  $P_I$  curve. It can be shown that  $P_I$  lies to the left of  $(g_1, d_{p1})$  for the parameters above<sup>9</sup> and the increase in wealth actually improves poor's consumption.

Figure A1 Here

Figure A2 Here

*Proof of Proposition 2*

The situation is depicted by Figure A3. A lower population share of the poor, rotates the  $P_{II}$  curve around  $(g_2, d_{p2})$ .

When the inequality is high enough the  $Q$  curve lies below  $(g_2, d_{p2})$ . A lower population share of the poor has the same effect on the  $Q$  curve as in the case  $0 < d_c \leq d_p$ . The shift is shown by the arrow  $(Q_{IIa} \rightarrow Q'_{IIa})$ . The shift in both curves implies that up to a certain level of  $d_p$  a lower population share of the poor increases growth. Same arguments apply as in the case when inequality

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<sup>9</sup>It turns out that  $(g, d_p)=(0.32, 0.18)$  and  $g_1 = 0.08$  in Figure 9

is extremely high. A lower population share of the poor increases the growth rate.

The difference here is the increase in the growth rate is diminishing as a result of two opposite effects. First, as the growth rate increases so does the interest rate. The future profits have to be discounted at a higher rate which reduces the entering firms' value. Less firms enter the market. This has a negative effect on growth. Second, the sales to the poor are realized faster due to improving purchasing power which increases profitability. The returns to innovation are therefore higher, which has a positive effect on growth. Moreover, a lower population share for the poor implies that rich's wealth position is worse. Less resources are spent on the production of luxuries improving efficiency and enhancing growth. The negative effect, however, dominates above a certain level of inequality as will be explained below.

An increase in wealth of the poor rotates the  $Q_{II}$  curve around  $(g_2, d_{p2})$  in Figure 10. It can be shown that P curve lies to right of  $(g_2, d_{p2})$  labeled as  $P_{IIa}$ . An increase in the wealth of the poor then unambiguously increases the growth rate when the inequality is high ( $g_{2a} \rightarrow g'_{2a}$ ). An increase in the wealth of the poor, increases the poor's consumption and speeds up the anticipated demand jump for the entering firms.

Figure A3 Here

*Proof of Proposition 3*

The situation is depicted again by Figure A3. A lower population share of the poor, rotates the  $P_{II}$  curve around  $(g_2, d_{p2})$ . The Q curve lies above  $(g_2, d_{p2})$  and is shown by  $Q_{IIb}$ . When the inequality is low, a lower population share of the poor reduces growth. A decrease in poor's population, when the inequality is sufficiently low, increases the demand for the luxuries ( $j > n$ ) more than it does the demand for new goods. In other words, when the poor can already afford most of the new goods a further decrease in inequality does not increase the entering firms' market size more than it does the incumbent, luxuries producers'. The production becomes less efficient and less

resources are diverted to R&D reducing growth.

As mentioned above an increase in the wealth of the poor does not have any effect on the  $P_{II}$  curve (Figure A4). Therefore, an increase in the wealth of poor when the initial inequality is low would still enhance growth. However, if the low inequality is coupled with a high initial consumption position of the poor or if the time preference is slightly higher (both shifting the P curve to the left) an increase in the wealth of the poor reduces both the growth rate and the poor's consumption. ( $g_{2b} \rightarrow g'_{2b}$ ). When the poor's consumption is already high, increasing the poor's wealth further shrinks the size of the entering firms' market by reducing the rich's consumption position  $d_r$  below 1. Entering firm's market shrinks even though it anticipates a larger market in the near future. A higher time preference tends to shift the P curve to the left as explained above.

*Proof Proposition 4*

A lower population share of the poor does not have any effect on growth or poor's consumption when the inequality is sufficiently small. Note that the last part of (14) are independent of the poor's population share,  $\phi$ . An increase in the wealth of the poor has no effect on the  $P_{III}$  curve in Figure A5. It rotates the Q curve counter-clockwise. Depending on the initial growth level an increase in the wealth of the poor may hurt or improve poor's consumption. Since the interest rates, given the time preference, are determined by the growth rate, with a low initial growth rate ( $g_{3a}$ ) increasing poor's wealth increases poor's consumption. In other words, it is optimal for the poor to save less. When the initial growth rate is high ( $g_{3b}$ ) the optimal consumption choice dictates that the poor save more. If the resources that are diverted to the production of luxuries in low-growth case are high enough the growth rate may decrease sufficiently enough so that the economy goes back to case 2 in the long run

Figure A4 Here

Figure A5 Here

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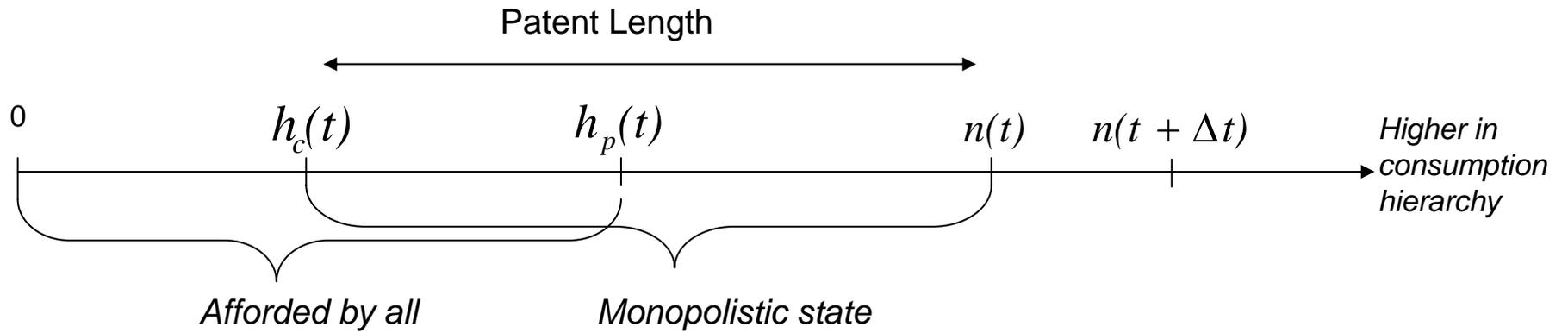


Figure 1. Inequality and Innovation Dynamics

Value of the Firm

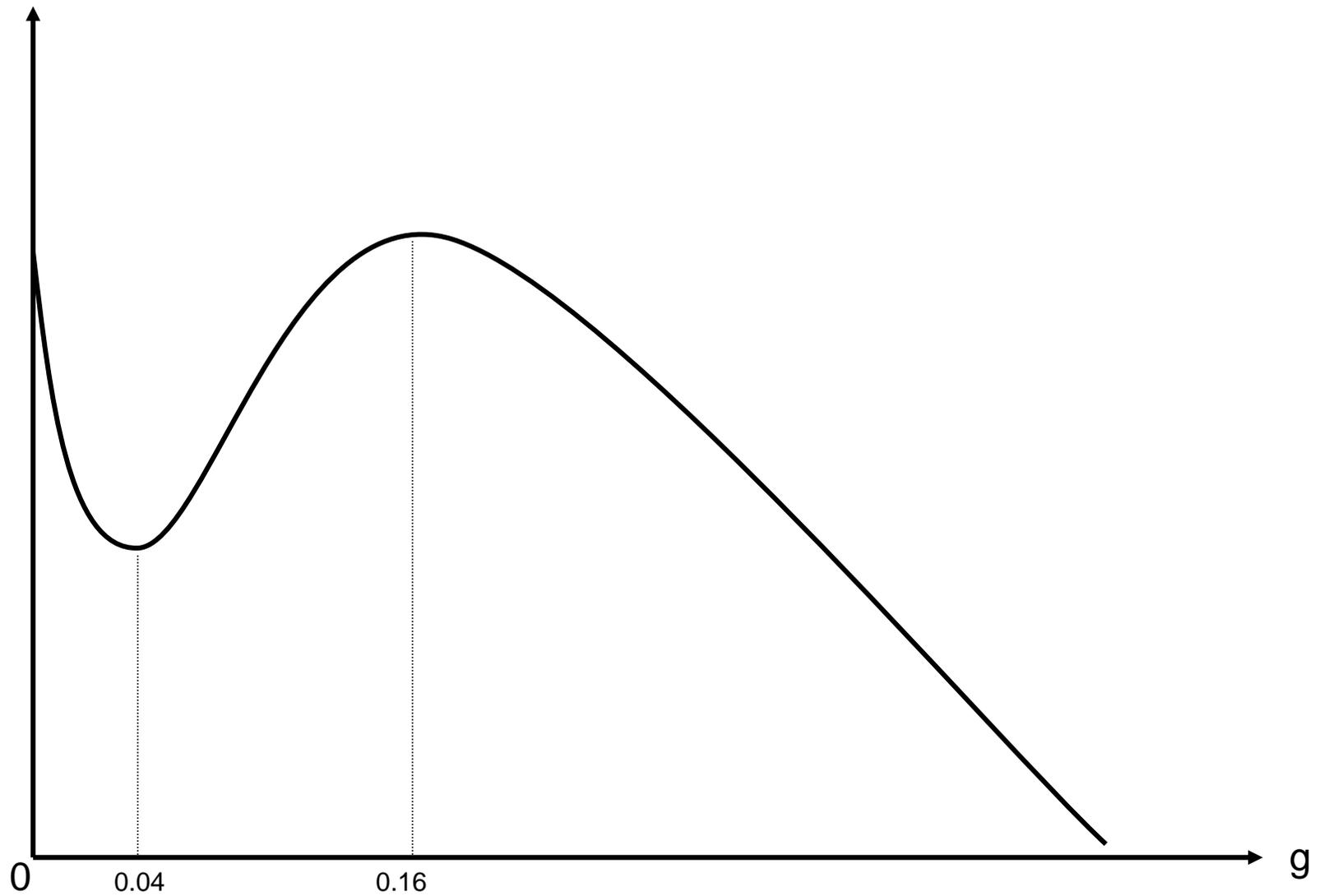


Figure 2. Value of a Monopolistic Firm.

Costs, Returns

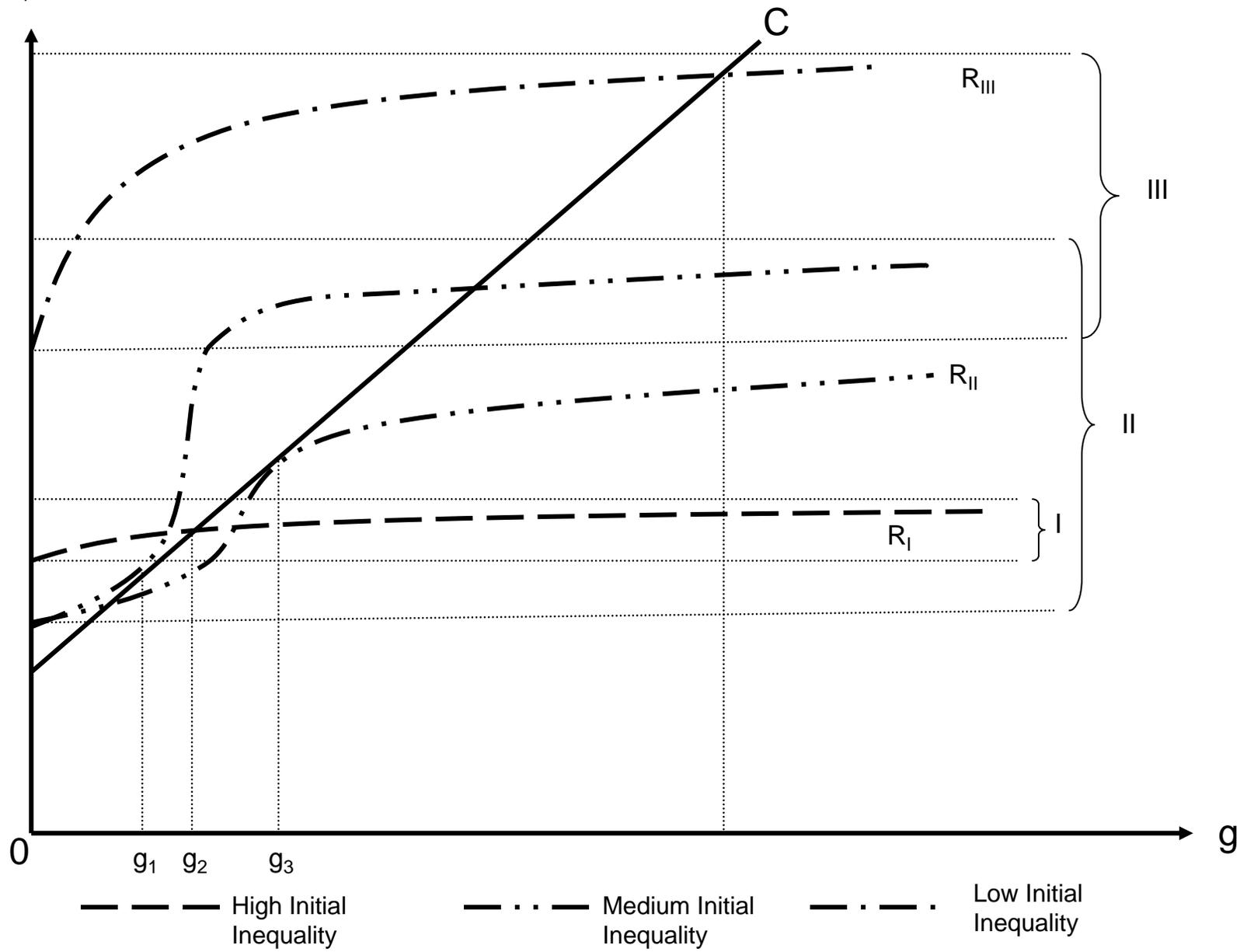


Figure 3. Partial Equilibrium

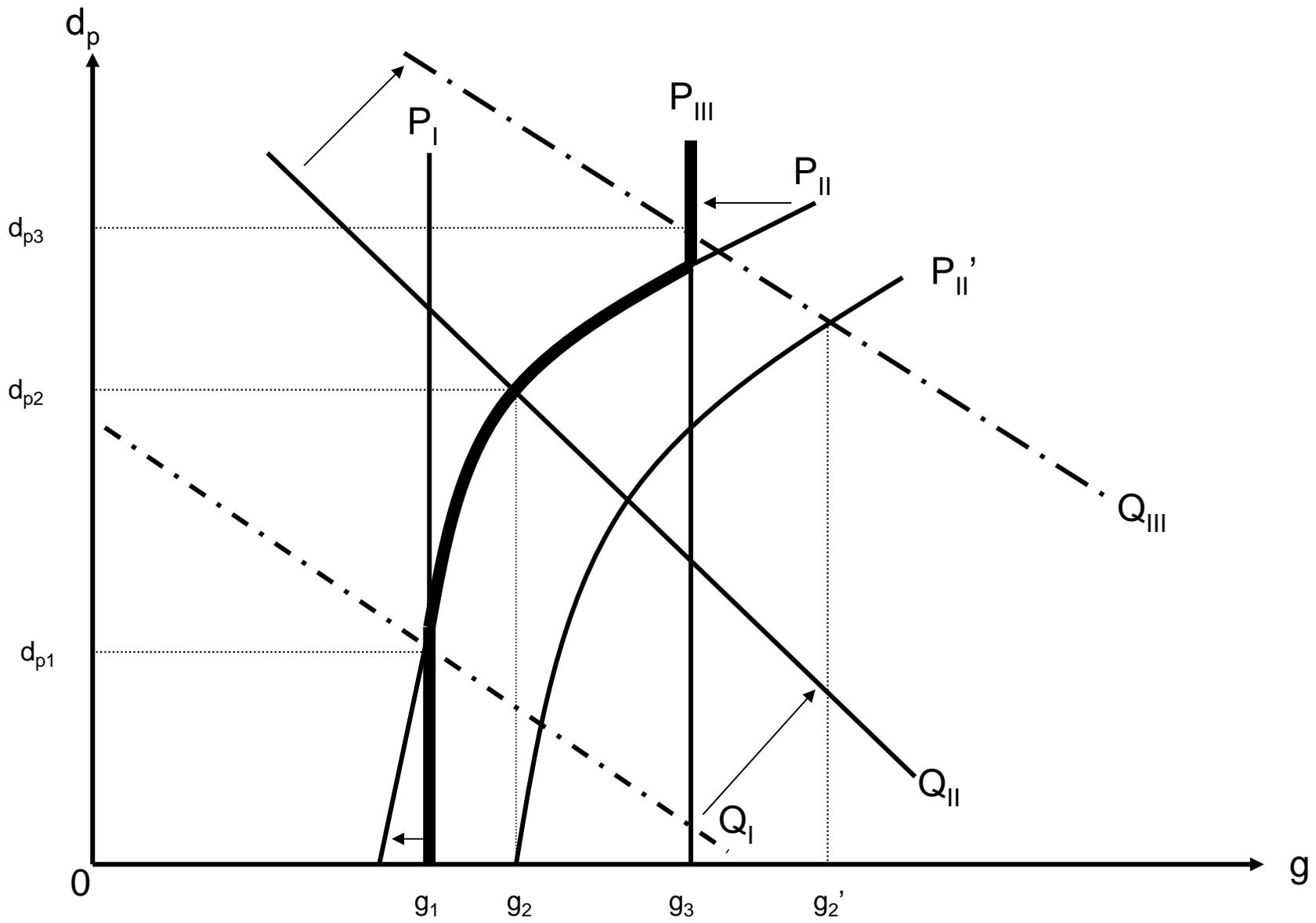


Figure 4. Set of Unique Equilibria.

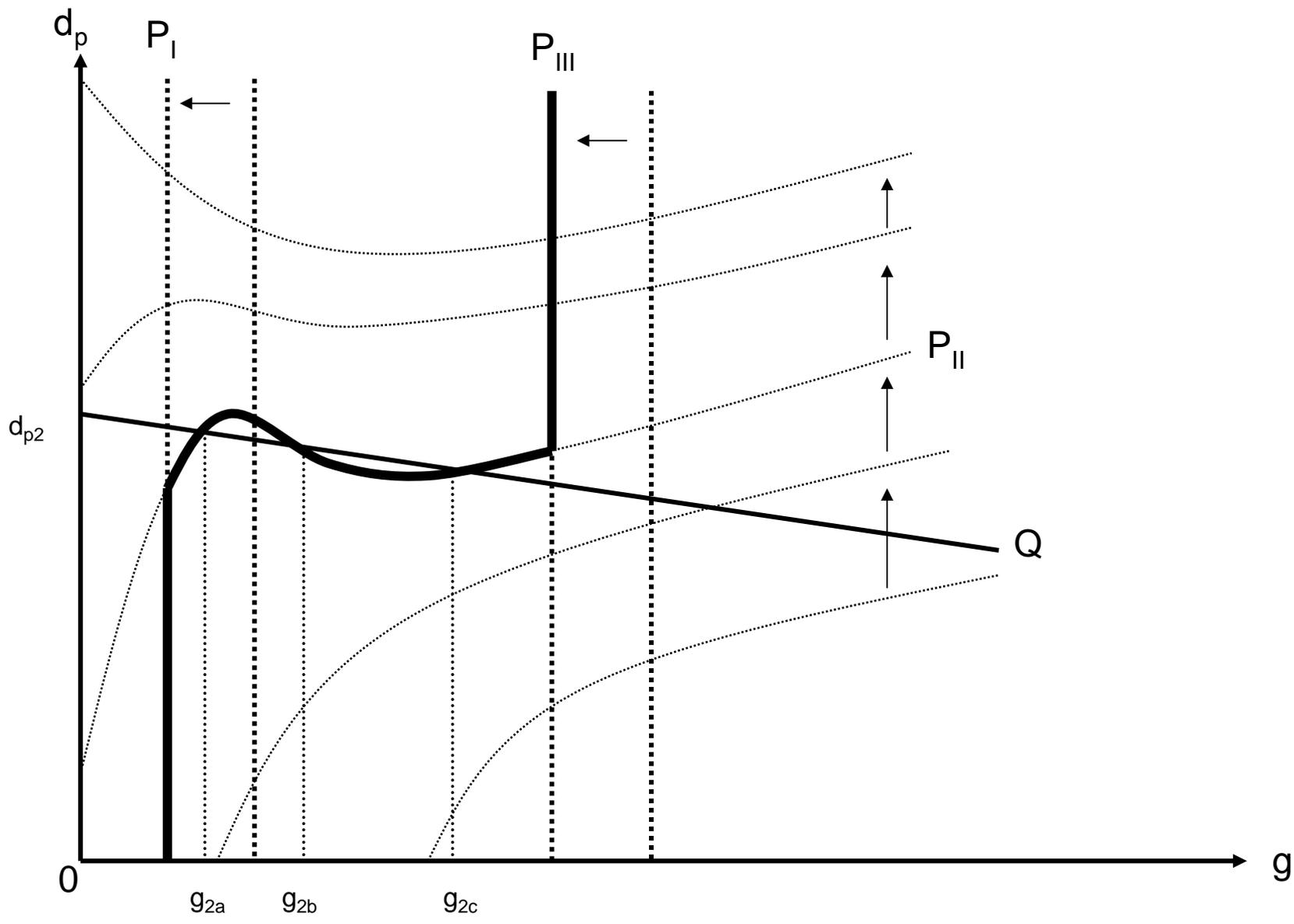


Figure 5. Multiple Equilibria.

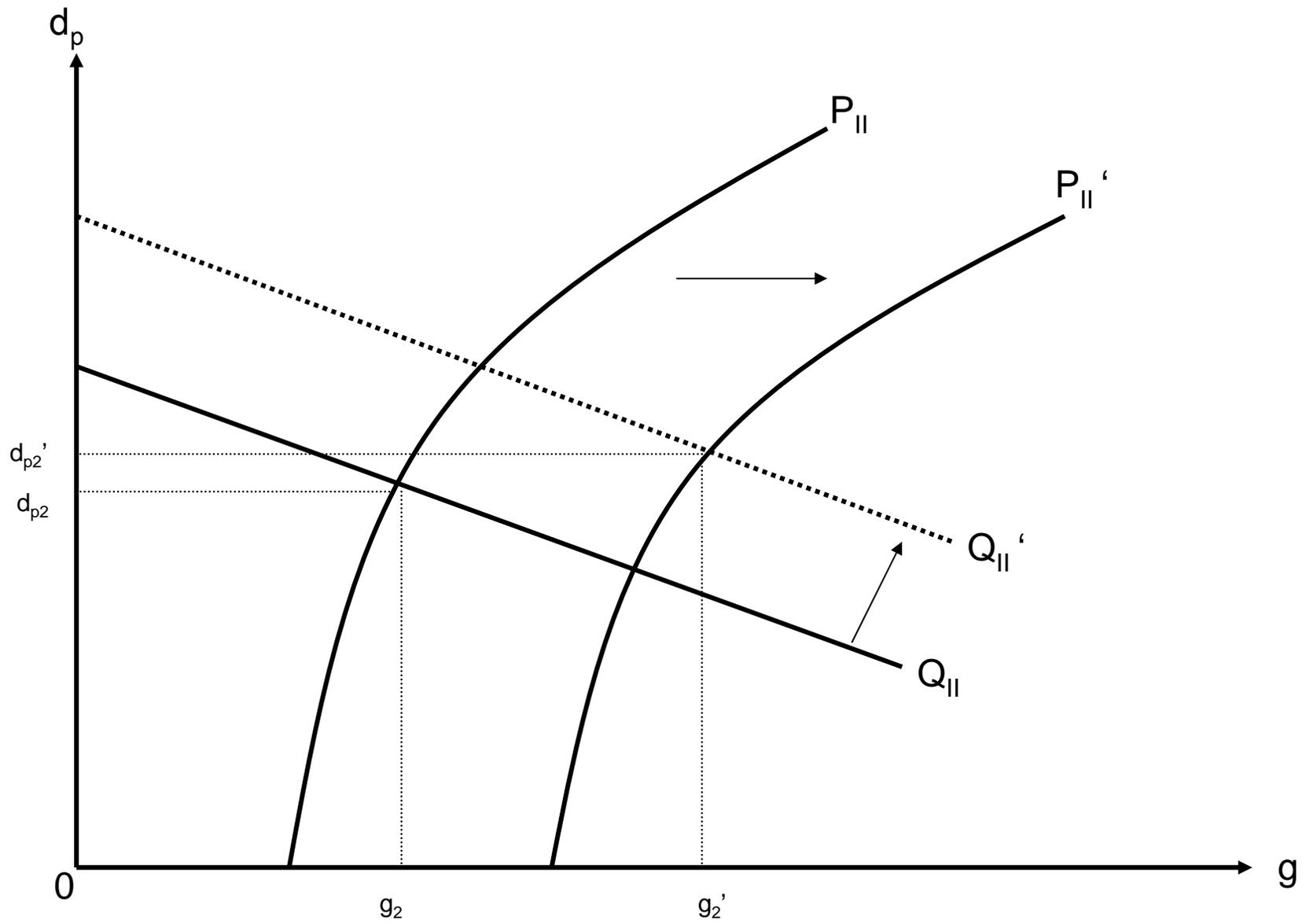


Figure 6. Increase in Patent Length.

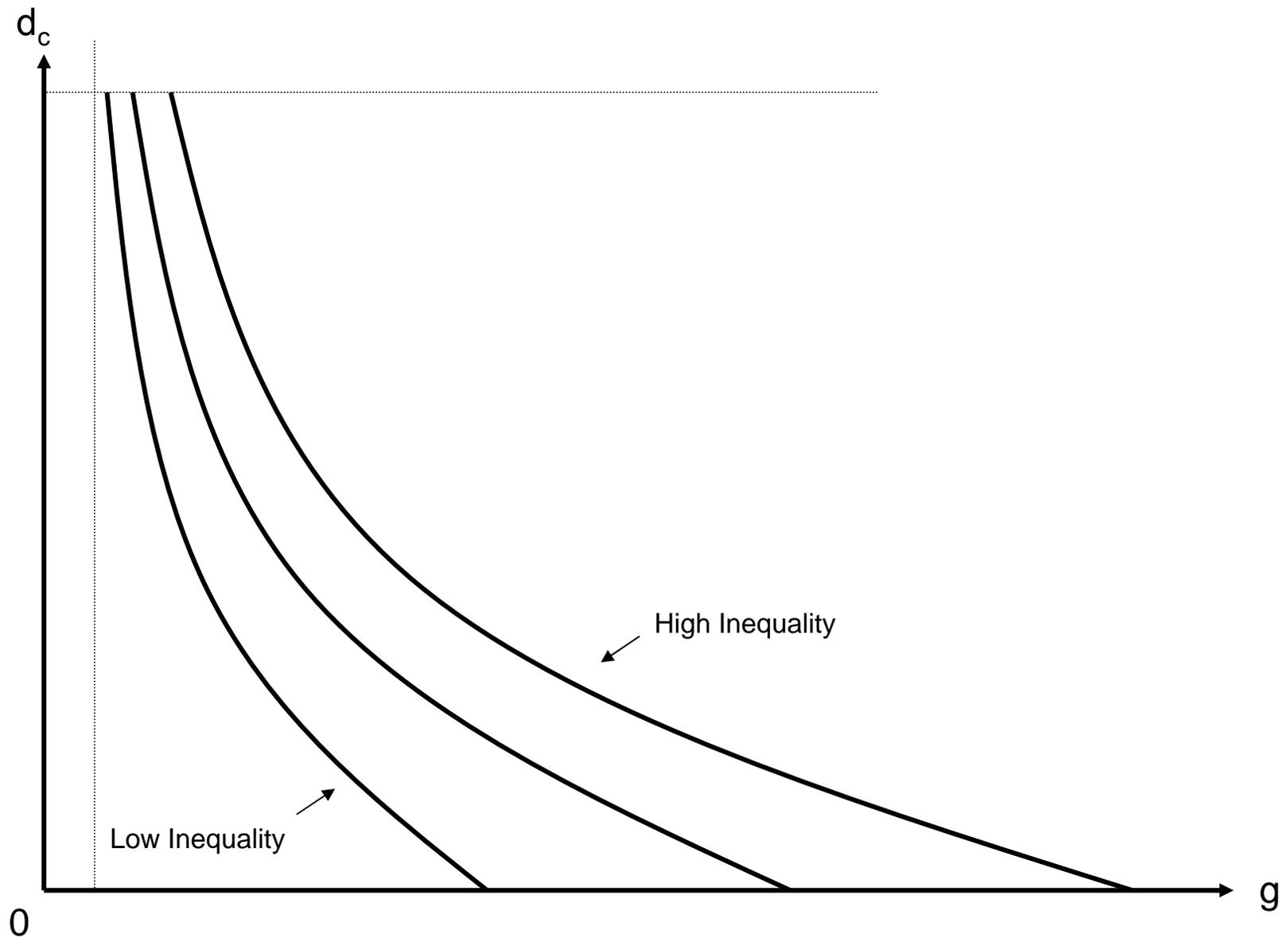


Figure 7. Patent Length and Growth. Effect of Inequality.

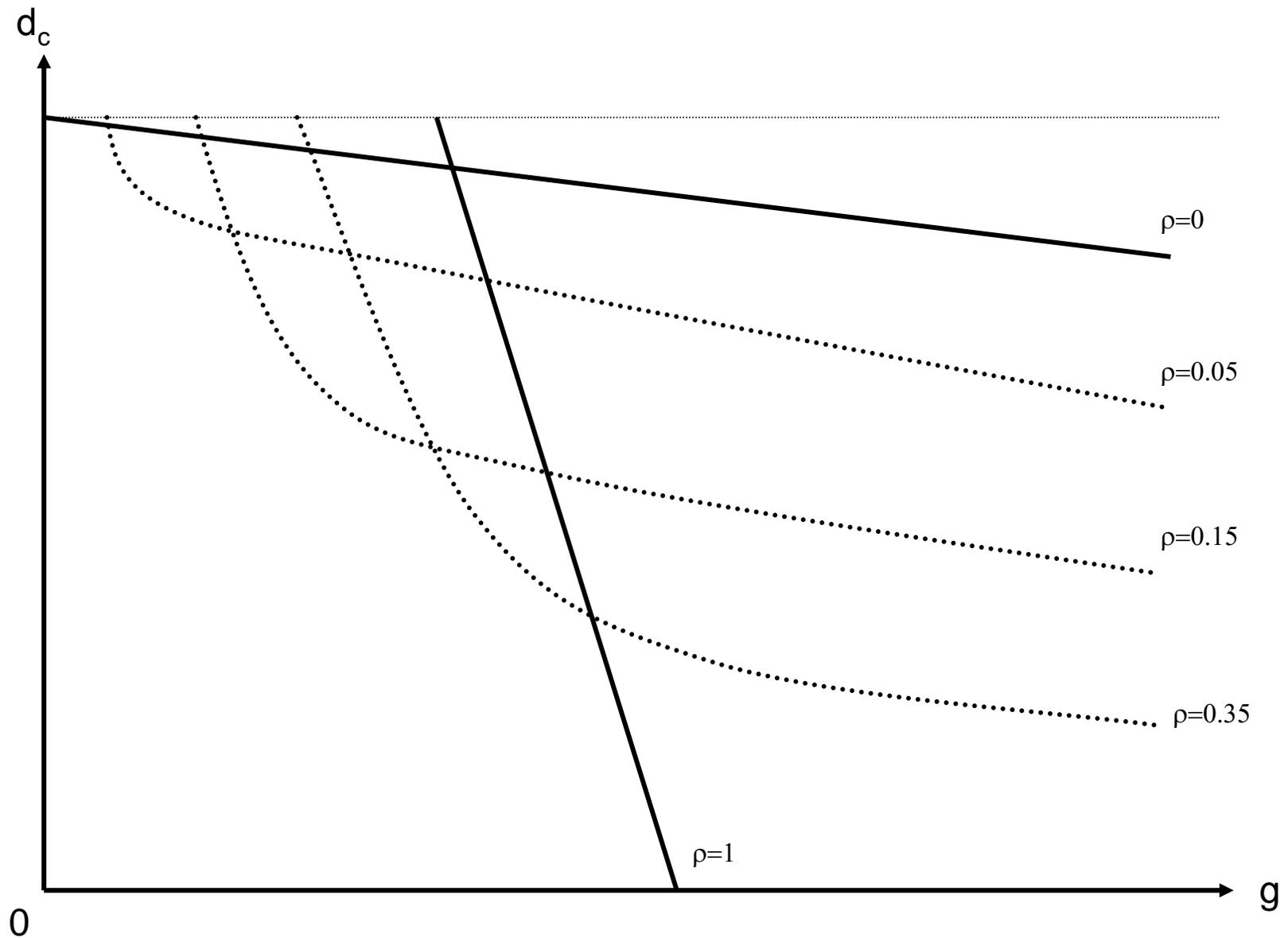


Figure 8. Patent Length and Growth. Effect of Time Preference.

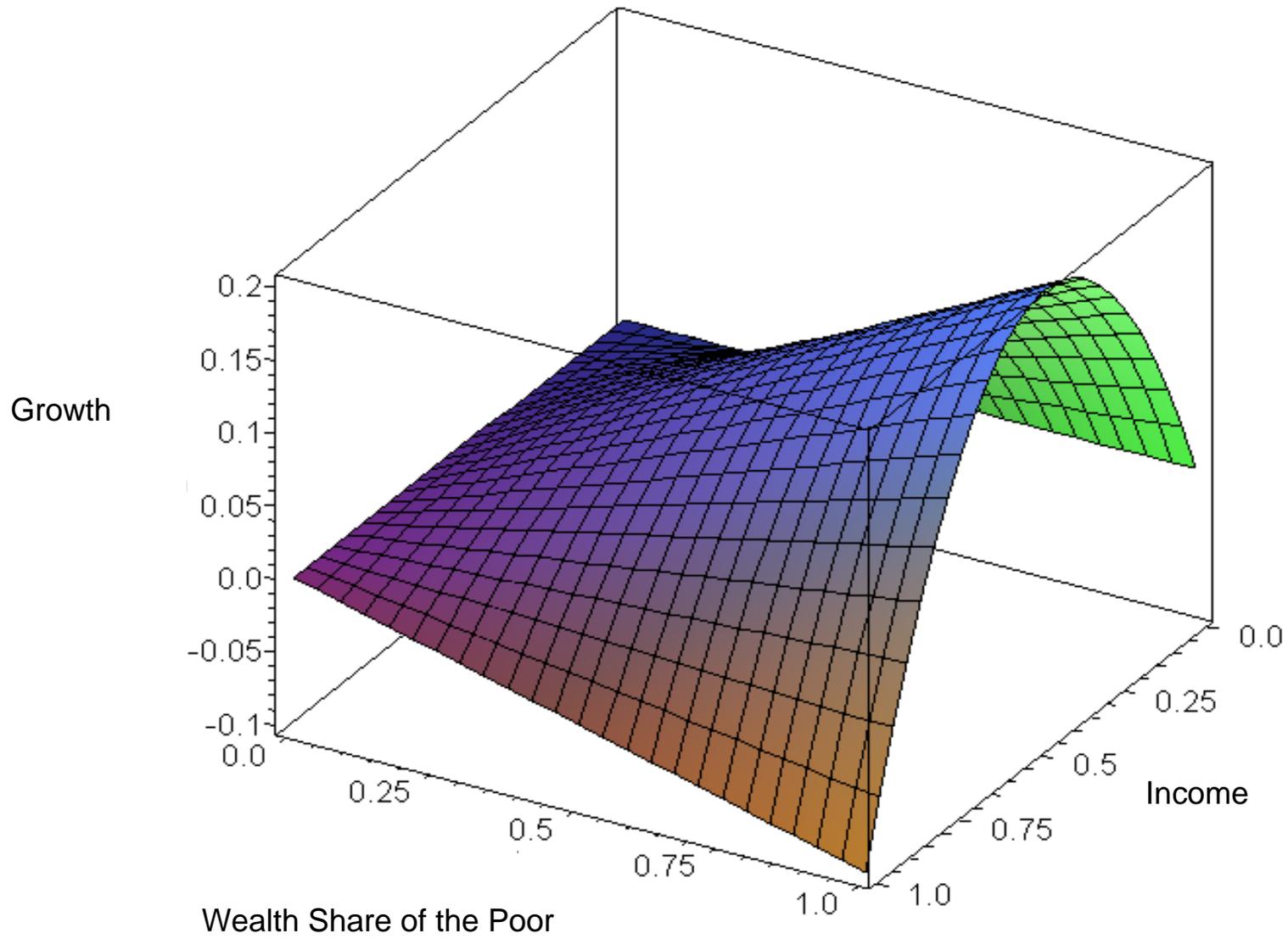
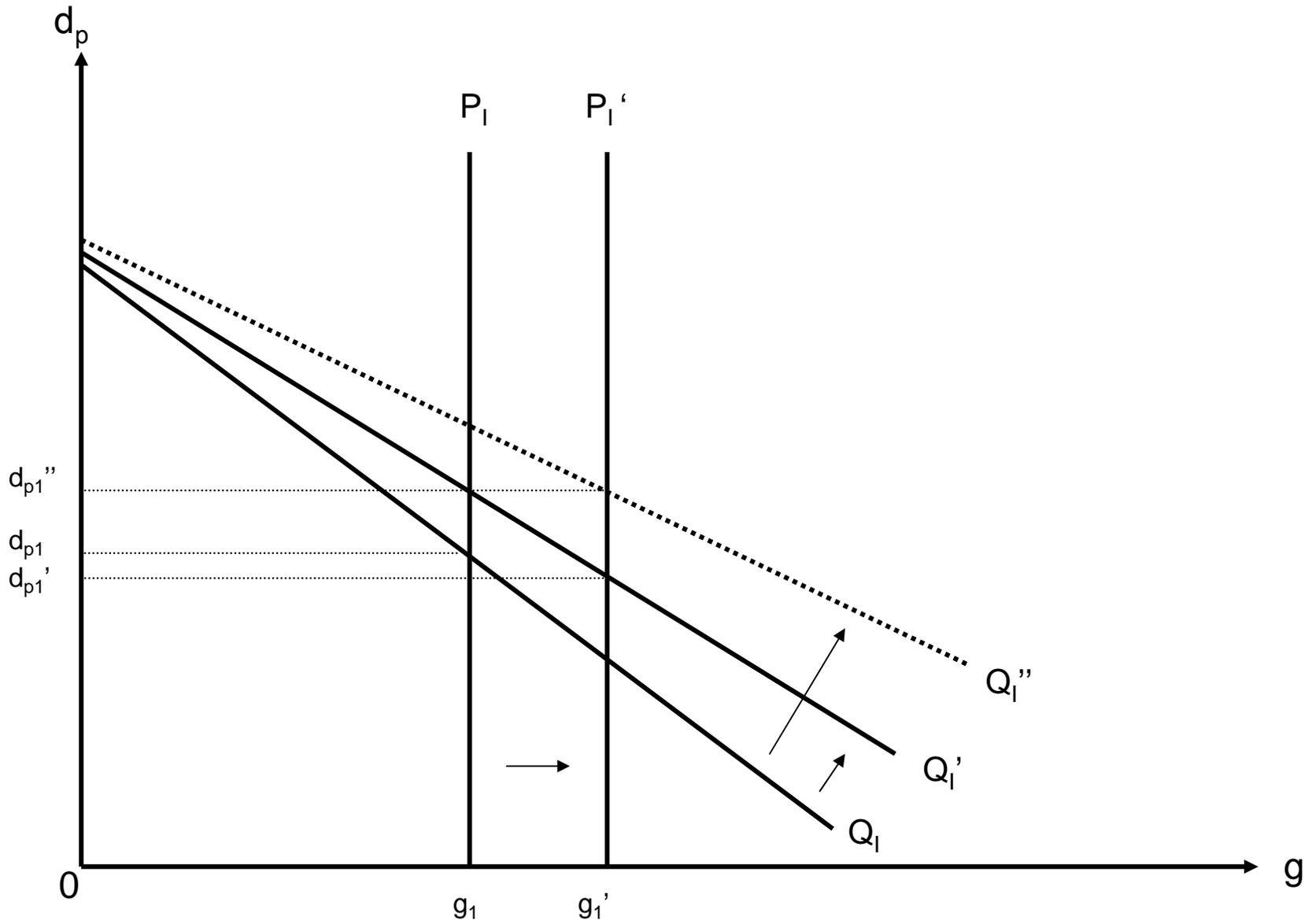
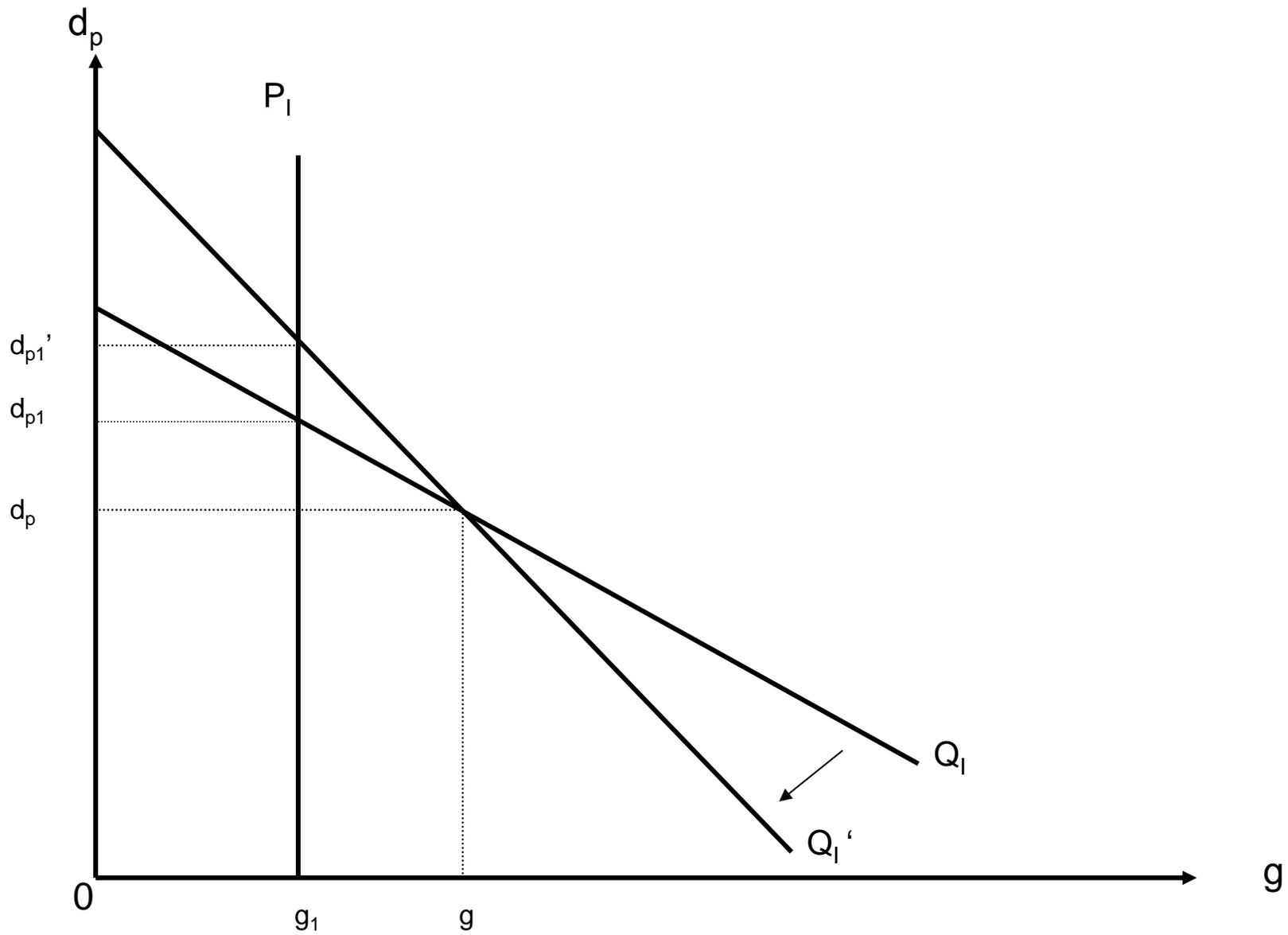


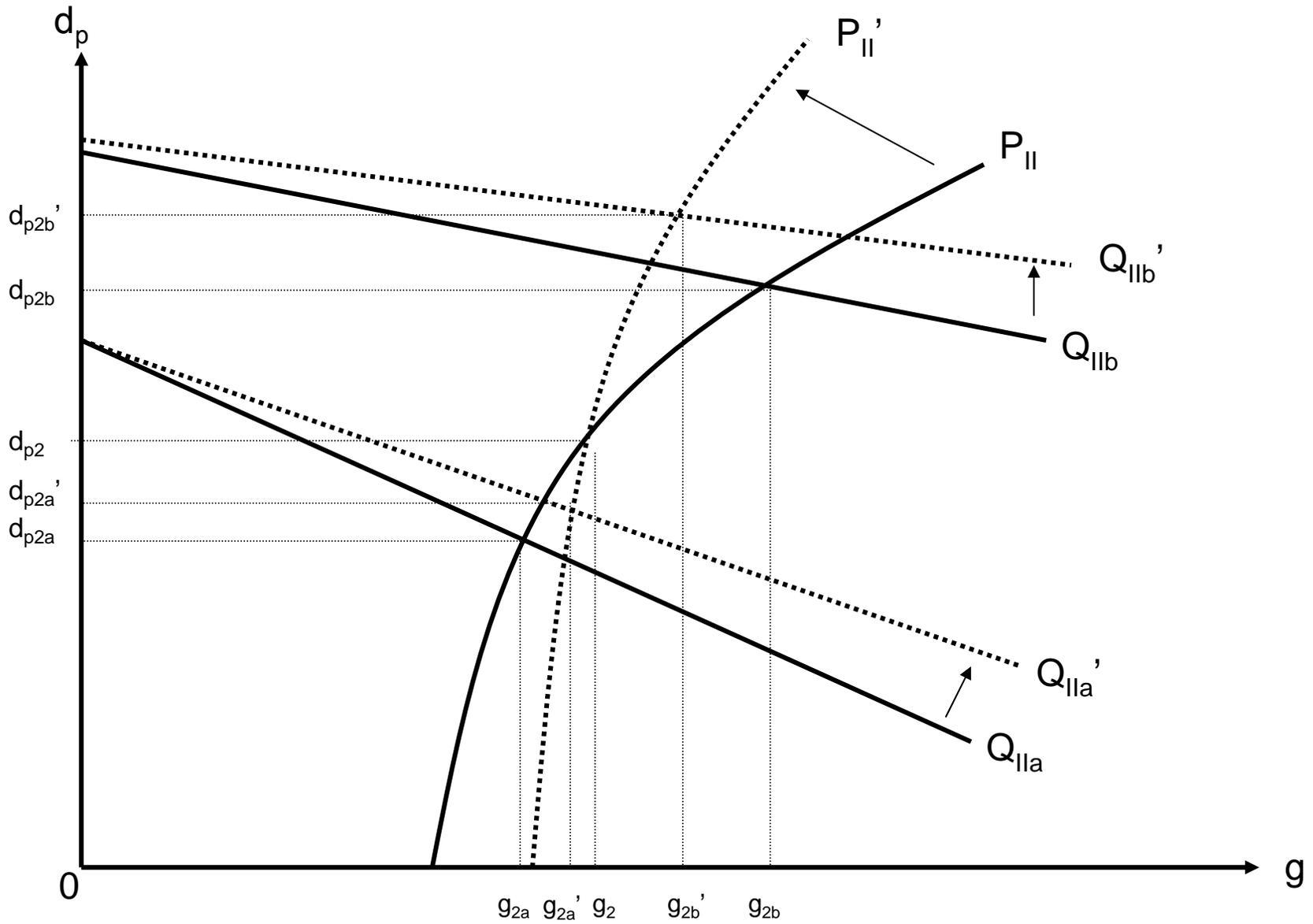
Figure 9. Inequality, Growth and Income.



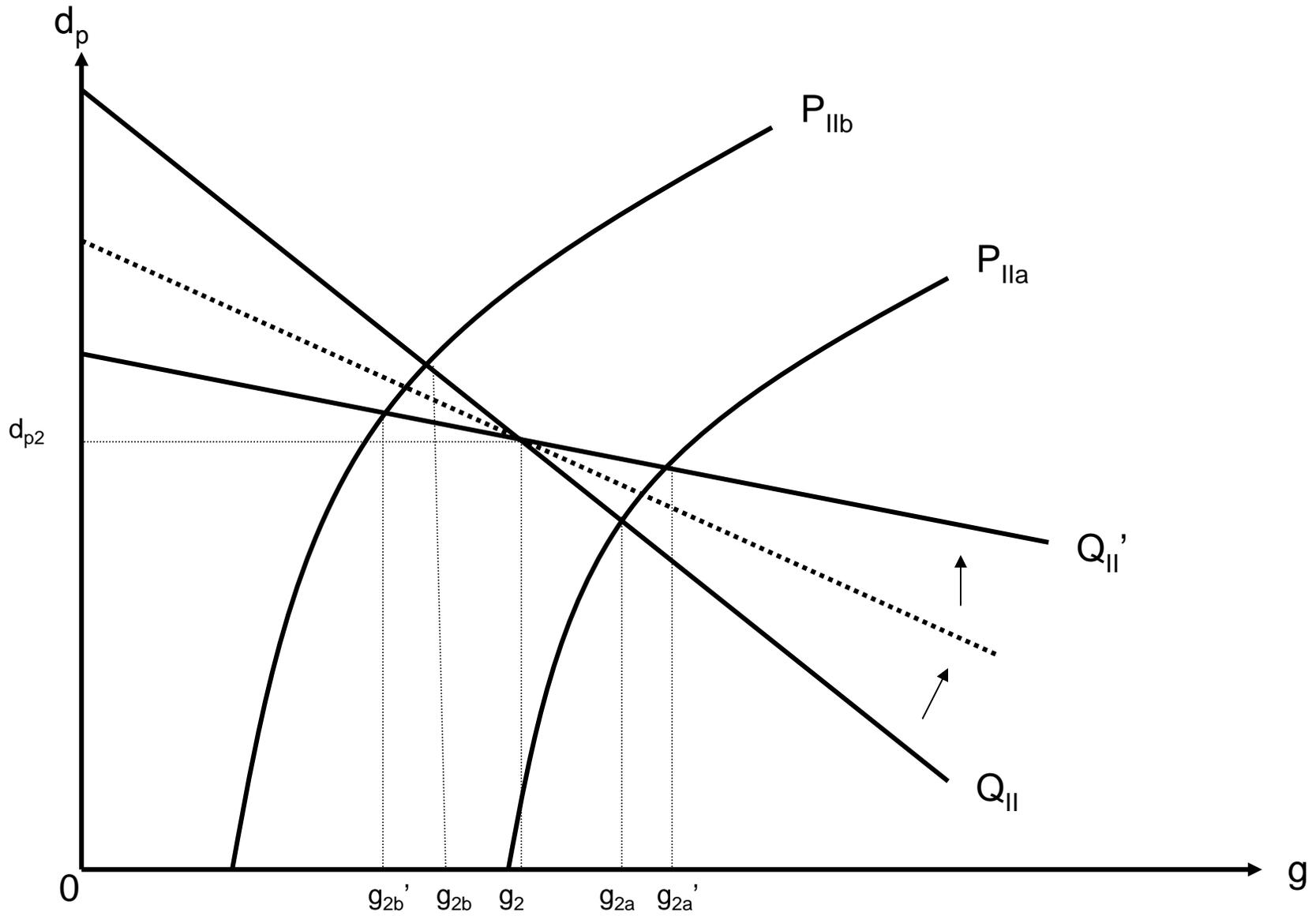
A1. A Lower Population Share of The Poor When Inequality Is Very High.



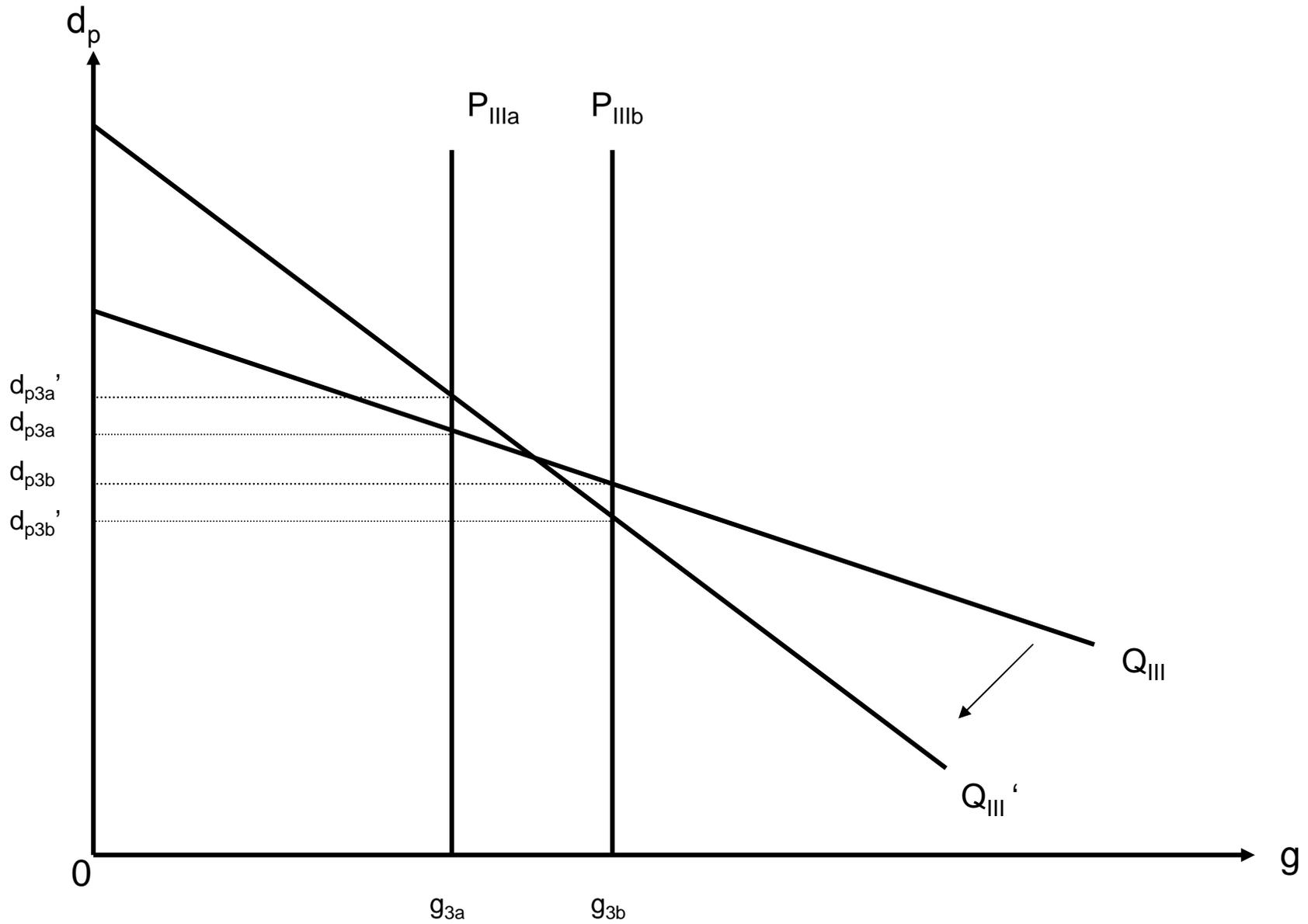
A2. An Increase in the Wealth of Poor When Inequality is Very High.



A3. A Lower Population Share of The Poor When Inequality is High.



A4. An Increase in the Wealth of Poor When Inequality is High.



A5. An Increase in the Wealth of Poor When Inequality is Very Low.