A Degree and an Efficiency of Manipulation of Known Social Choice Rules

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Abstract. 26 known and new social choice rules are studied via computational experiments to reveal to which extent these rules are manipulable. 5 indices of manipulability are considered.

Keywords. Social choice rules, degree of manipulation, efficiency of manipulation

Introduction

Manipulation is a phenomenon observed in collective decision making. It deals with the intentional deviation of an agent's preference about considered alternatives which leads to the better social choice for that very agent.[1]

Let seven members of a group have the following preferences about three alternatives[2]: for 3 agents -- $xPyPz$; for 2 agents -- $yPzPx$; and for 2 agents -- $zPyPx$ Then, if the plurality rule is used, x will be chosen, though it is the worst alternative for the majority (four) of the agents. In such a situation if the agents from the third group change their preferences to $yPzPx$, i.e. put on top their second best alternative y, then according to the rule the alternative y will be chosen with 4 votes. So, the intentional deviation of the true preferences of the agents from the third group leads to a more desirable result - instead of the worst alternative x, the alternative y is chosen.

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[1] Such type of manipulation is called manipulation by an agent. Other types of manipulation are also known, e.g., agenda manipulation.

[2] Preferences are supposed to be transitive: if $xPyPz$, then xPz.

Theoretical investigation of manipulation problem shows that in a very wide framework any rule transforming agents' preferences into social choice is either manipulable or dictatorial (Gibbard(1973), Satterthwaite(1975)). To which extent social choice correspondences are manipulable were studied by Kelly(1993), Smith(forthcoming). More concrete, Kelly(1993) introduced the degree of manipulability of choice procedures. A degree of manipulability was measured as the ratio of manipulable profiles, i.e., profiles, in which at least one agent can manipulate, to the total number of profiles. Unlike Kelly(1993), in Smith(forthcoming) already several measures of manipulation were studied. The present work continues the articles by Kelly(1993), Smith(forthcoming). We investigate a degree and an efficiency of manipulation of 26 known and new social choice procedures, including those which were studied in Smith(forthcoming). The articles Kelly(1993), Smith(forthcoming) and present paper also have another common feature. Since the problem of theoretical investigation of degree of manipulability of social choice rules has not been solved yet, in present work analysis has been done by computer computations. The computations have been carried out in two ways. With small number of agents n and alternatives m the computations were exhaustive, i.e., included complete examination of manipulability of all profiles. In those cases when such examination required great amount of time (for some rules already with $n = 5$ and $m = 4$), statistical method have been applied.

The structure of the paper is as follows. In Section 2 measures of manipulability are described. In Section 3 26 social choice rules under study are given. In Section 4 computation scheme is given. In Section 5 the results are discussed and concluding remarks are given.

Tables 1-5 contain the results of calculations of 5 indices of manipulability for 26 social choice rules. Some of the results presented can be found in Aleskerov and Kurbanov(1997)

Since the corresponding software is ready and had been very carefully tested, all suggestions to evaluate new rules or to check new criteria of manipulability are welcome.

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The Framework

The following problem is considered. A finite set A of alternatives is given which consists of m alternatives ($m > 2$). Agents from finite set $N = \{1,\dots,n\}$ $(n > 1)$, have preferences over alternatives from the set A. This preferences are assumed to be linear orders , that is

- irreflexive $(x\overline{P}x)$,
- transitive ($\forall x,\ y,\ z\ \ xPy \,\&\, yPz \Rightarrow xPz$), and
- connected ($\forall x, y$ either xPy or yPx) binary relations.

An ordered n-tuple of individual binary relations is called a profile. Social decision is considered to be an element of the set A. The set of all linear orders on A is denoted as $\Re$. Then social choice rule is defined as $C: \Re^n \to A$

The effect of manipulability can be described as follows. Let $\vec{P} = \left(P_1,\dots,P_i,\dots,P_n\right)$ be the profile of agents' true preferences, while $\vec{P}_{-i} = \left(P_1,\dots,P_{i-1},P_i',P_{i+1},\dots,P_n\right)$ be a profile in which all agents but i-th reveal their true preferences. Let $C(\vec{P})$, $C(\vec{P}_{-i})$ denote (single valued) social choice with respect to the profile $\vec{P}$ and the profile $\vec{P}_{-i}$, respectively. Then $C(\vec{P}_{-i})P_iC(\vec{P})$, that is the outcome when i-th agent deviates from her true preference is more preferable for her than in the case when she expresses her sincere opinion.

We study the effect of manipulability under the condition that social choice always consist of only one alternative. Hence the following common way how from multiple choice (which is defined by the rule under the given $\vec{P}$) switch to a single valued choice is accepted. In the case of multiple choice the outcome has been chosen with respect to the alphabetical order of chosen alternatives. For instance, if the choice is $C(\vec{P}) = \{b, d, x\}$, then final social choice is defined to be b.

Indices of manipulability

Number of alternatives being m, the total number of possible linear orders is obviously equal to $m!$, and total number of profiles with n agents is equal to $(m!)^n$. In Kelly(1993) to measure a degree of manipulability of social choice rules the following index was introduced (we call it Kelly's index and denote as K) :

$$K=\frac{d_0}{(m!)^n} \ ,$$

where d_0 is the number of profiles in which manipulation takes place.

We also studied an extended version of Kelly's index. Denote by l_k the number of profiles in which exactly k voters can manipulate. Construct index $J_k=\frac{l_k}{(m!)^n}$ which shows the share of profiles in which exactly k voters can manipulate. Obviously, $K=J_1+J_2+\ldots+J_n$. Then we consider vectorial index $J=(J_1,J_2,\ldots,J_n)$.

Before defining the next criterion of manipulability let us note that for an agent there are $(m!-1)$ linear orders to use instead of her sincere preference. But in how many cases her manipulation would be successful? The answer on this question leads to the notion of "freedom" of manipulation of a given agent in a given profile. Denote as X_i^j $(i=1,\ldots,n;\ 0\le X_i\le m!-1)$ the number of such orderings in the profile j. Dividing it to $(m!-1)$ one can find the freedom of manipulation of an agent i in this profile. Summing up this index over all agents and dividing it to n one can find the average freedom of manipulation in the given profile. Summing the index over all profiles and dividing this sum to $(m!)^n$ we obtain our next index

$$I_1=\frac{\sum_{j=1}^{(m!)^n}\sum_{i=1}^{n}X_i^j}{(m!)^n\cdot n\cdot(m!-1)}.$$

Thus, the index I_1 defines an average freedom of manipulation of a given social choice function.

These indices K and I_1 (as well as index J) measure the degree of manipulability in terms of the share of manipulable profiles or the share of orderings using which an agent can manipulate.

The following two indices show the *efficiency* of manipulation, i.e., to which extent an agent can be better off via manipulating her sincere ordering. Let under a profile $\vec{P}$ social decision be an alternative x which stands at k-th place from the top in the sincere ordering of i-th agent. Let after her manipulation the social decision be an alternative y which stands in the sincere ordering of the i-th agent at j-th place from the top, and let $j<k$. Then $q=k-j$ shows how is the i-th agent better off . Let us sum up q for all advantageous orderings X_i^j for profile j (defined above), and let us divide the obtained value to X_i^j. Denote this index through Z_i^j, which shows an average "benefit" of manipulation of the agent i gained via manipulation X_i^j orderings from $(m!-1)$. Summing up this index over all agents and over all profiles, we obtain the next index under study

$$I_2=\frac{\sum_{j=1}^{(m!)^n}\sum_{i=1}^{n}Z_i^j}{(m!)^n\cdot n}.$$

The indices I_1 and I_2 were used in Smith(forthcoming) as well.

The next criterion I_3 is a modification of I_2. Instead of evaluating the "average" benefit Z_i^j for i -th agent in profile j, we evaluate the value

$$Z_{ij}^{\max} = \max(Z_1, Z_2, \ldots, Z_{X_i^j}).$$

In other words, the value $Z_{ij}^{\max}$ show the maximal benefit which can be obtained by agent i in profile j. Summing up this index over all agents and over all profiles, we obtain our next index under study

$$I_3 = \frac{\sum_{j=1}^{(m!)^n} \sum_{i=1}^{n} Z_{ij}^{\max}}{(m!)^n \cdot n}.$$

The indices K, J, I_1, I_2, I_3 have been calculated for each of the rules introduced in the next section.

Social choice rules

Let us introduce several important notions and definitions, which are permanently used hereafter. The concepts and rules used below can be found in Aleskerov(1985), Aleskerov(1992), Arrow (1963), Banks(1985), Brams and Fishburn(1981), Moulin(1987), Nurmi(1987), Smith (forthcoming), Vol'sky and Lezina(1991).

Definition 1. Majority relation for a given profile $\vec{P}$ is a binary relation m which is constructed as follows

$$xm\, y \Leftrightarrow card\{i \in N \mid xP_i y\} > card\{i \in N \mid yP_i x\}$$

Definition 2. Condorcet winner $CW(\vec{P})$ in the profile $\vec{P}$ is an element undominated in the majority relation m (constructed according to the profile), i.e.,

$$CW(\vec{P}) = \{a \mid \overline{\exists} x \in A, xm\, a\}$$

Definition 3. A contruction of a profile $\vec{P}$ onto the set $X \subseteq A$, $X \neq \varnothing$ is a profile

$$\vec{P}/X = (P_1/X, \ldots, P_n/X),\ P_i/X = P_i \cap (X \times X)$$

Definition 4. Upper counter set of an alternative x in the relation P is the set $D(x)$ such that

$$D(x) = \{y \in A \mid yPx\}$$

Lower counter set of x in the relation P is the set $L(x)$ such that

$$L(x) = \{y \in A \mid xPy\}$$

Let's describe now social choice rules. The rules under study can be divided into several groups.

a) Scoring Rules;

b) Rules, using majority relation;
c) Rules, using value function;
d) Rules, using tournament matrix;
e) q-Paretian rules.

Scoring Rules

1. Plurality Rule.

Choose alternatives, that have been admitted to be the best by the maximum number of agents, i.e.

$$a \in C(\vec{P}) \Leftrightarrow \left[\forall x \in A \quad n^+(a, \vec{P}) \geq n^+(x, \vec{P})\right],$$

$$n^+(a, \vec{P}) = card\left\{i \in N \,\middle|\, \forall y \in A \; aP_i y\right\}$$

Consider the example with 5 agents and 4 alternatives given below.

P_1	P_2	P_3	P_4	P_5
a	b	d	b	a
d	a	a	c	c
b	c	b	a	d
c	d	c	d	b

In this example according to the plurality rule alternatives a and b will be chosen. Then, the single-valued choice will be a .

2. Approval Voting.

Let us define

$$n^+(a, \vec{P}, q) = card\left\{i \in N \,\middle|\, D_i(a) \leq q\right\}$$

i.e., $n^+(a, \vec{P}, q)$ means the number of agents for which a is placed on $q+1$'st place in their orderings. Thus, if $q = 0$, then a is the first best alternative for i -th voter; if $q = 1$, then a is either first best or second best option, etc. The integer q can be called as degree of procedure.

Now we can define Approval Voting Procedure with degree q

$$a \in C(\vec{P}) \Leftrightarrow \left[\forall x \in A \quad n^+(a, \vec{P}, q) \geq n^+(x, \vec{P}, q)\right],$$

i.e., the alternatives are chosen that have been admitted to be between $q+1$ best by the maximum number of agents.

It can be easily seen that Approval Voting Procedure is a direct generalization of Plurality Rule; for the latter $q \equiv 0$.

Remark. Approval Voting Procedure is in general defined in such a way that a voter can choose any number of alternatives from the top, i.e., q is defined not for all voters equally (Brams and Fishburn (1981)) .

3. Run-off Procedure.

First, simple majority rule is used (an alternative with 50%+1 votes will be chosen). If such an alternative exists the procedure stops; otherwise, two alternatives with maximal number of votes are taken. Assuming that preferences of agents about these two alternatives do not change, again simple majority rule is applied. Since agents' preferences are linear orders single winner (if number of agents is odd) always exists.

4. Hare's Procedure.

Firstly simple majority rule is used. If such alternative exists, the procedure stops; otherwise, the alternative x with the minimum number of votes is omitted. Then the procedure again applied to the set $X = A \setminus \{x\}$ and the profile $\vec{P}/X$.

5. Inverse Plurality Rule.

The alternative, which is regarded as the worst by the minimum number of agents, is chosen, i.e.,

$$a \in C(\vec{P}) \Leftrightarrow \left[\forall x \in A \quad n^{-}(a, \vec{P}) \le n^{-}(x, \vec{P})\right],$$
$$n^{-}(a, \vec{P}) = card\left\{i \in N \,\middle|\, \forall y \in A \;\, yP_i a\right\}.$$

Consider the previous example. In this example $\forall x \in A \;\; n^{-}(a, \vec{P}) = 0 \le n^{-}(x, \vec{P})$. Therefore $C(\vec{P}) = \{a\}$.

6. Borda's Rule.

Put to each $x \in A$ into correspondence a number $r_i(x, \vec{P})$ which is equal to the cardinality of the lower contour set of x in $P_i \in \vec{P}$, i.e. $r_i(x, \vec{P}) = card\big(L_i(x)\big)$. The sum of that numbers over all i is called Borda's count for alternative x.

Alternative with maximum Borda's count is chosen., i.e.

$$a \in C(\vec{P}) \Leftrightarrow \left[\forall b \in A, \; r(a, \vec{P}) \ge r(b, \vec{P})\right], \quad r(a, \vec{P}) = \sum_{i=1}^{n} r_i(a, P_i)$$

7. Black's Procedure.

If Condorset winner exists, it is to be chosen. Otherwise, Borda's Rule is applied.

8. Inverse Borda's Procedure.

For each alternative Borda's count is calculated. Then the alternative a with minimum count is omitted. Borda's count are re-calculated for profile $\vec{P}/X$, $X = A \setminus \{a\}$, and procedure is repeated until choice is found.

9. Nanson's Procedure.

For each alternative Borda's count is calculated. Then average count is calculated, $\bar{r} = \left(\sum_{a \in A} r(a, \vec{P}) \right) / |A|$, and alternatives $c \in A$ are omitted for which $r(c, \vec{P}) < \bar{r}$. Then the set $X = \left\{ a \in A \middle| \; r(a, \vec{P}) \geq \bar{r} \right\}$ is considered, and the procedure applied to the profile $\vec{P} / X$. Such procedure is repeated until choice will not be empty.

10. Coombs' Procedure.

Alternative which is the worst for maximum number of agents is omitted. Then profile is contracted to the set X, and the procedure is repeated until choice will not be empty.

Rules, using majority relation

11. Minimal dominant set.

A set Q is called dominant one if each alternative in Q dominates each alternative outside Q via majority relation. Otherwise speaking $\forall x \in A$

$$x \in Q \Leftrightarrow \left[\forall y \in A \setminus Q, \;\; xm\, y \right]$$

Then a dominant set Q is called the minimal one if none of it's proper subsets is a dominant set. Social choice if defined as $C(\vec{P}) = Q$. If such set is not unique, then the social choice is defined as the union of these sets. The final choice will consist of the alternative which is the first in alphabetical order.

12. Minimal Undominated Set.

A set Q is called undominated one if no alternative outside Q dominates some alternative in Q via majority relation. Undominated set Q is called the minimal one if none of it's proper subsets is undominated set. Social choice if defined as $C(\vec{P}) = Q$. If such set is not unique, then the social choice is defined as the union of these sets. The final choice will consist of the alternative which is the first in alphabetical order.

13. Minimal Weakly Stable Set.

A set $Q \subseteq A$ is called weakly stable if it has the following property: $\forall x \in A$

$$x \in Q \Leftrightarrow \left(\exists y \in A \setminus Q \; s.t. \; ymx \Rightarrow \exists z \in Q \; s.t. \; zm\, y \right),$$

i.e., x belongs to Q iff (if there exists an alternative $y \in A \setminus Q$ which dominates x via majority relation m, then there exists an alternative $z \in Q$, which dominates y, i.e., $zm\, y$). A set $Q \subseteq A$ is called the minimal weakly stable one if none of it's proper subsets is weakly stable set. Social choice if defined as $C(\vec{P}) = Q$. If such set is not unique, then the social choice is defined as the union of these sets. The final choice will consist of the alternative which is the first in alphabetical order.

This rule is an ordinal counterpart of Mashler's bargaining set (Mashler(1992)).

14. Fishburn's Rule.

Construct upper contour set $D(x)$ of x in the relation m, and the binary relation g as follows:

$$xg\ y \Leftrightarrow D(x) \subset D(y).$$

Then undominated alternatives on g are chosen[3], i.e.,

$$x \in C(\vec{P}) \Leftrightarrow \left[\,\overline{\exists}\, y \in A \,\middle|\, yg\ x\right].$$

15. Uncovered Set I.

Construct lower contour set $L(x)$ of x in the relation m, and the binary relation d as follows:

$$xd\ y \Leftrightarrow L(x) \supset L(y).$$

Then undominated alternatives on d are chosen, i.e.,

$$x \in C(\vec{P}) \Leftrightarrow \left[\,\overline{\exists}\, y \in A \,\middle|\, yd\ x\right].$$

16. Uncovered Set II.

An alternative x is said to B-dominate an alternative y (denoted as xBy) if $xm\,y$ and $D(x) \subseteq D(y)$, where $D(x)$ is the upper contour set of x in m. Social choice consists of B-undominated alternatives, i.e.,

$$x \in C(\vec{P}) \Leftrightarrow \left[\,\overline{\exists}\, y \in A \,\middle|\, yBx\right].$$

17. Richelson's Rule.

Construct lower and upper contour sets $D(x)$ and $L(x)$ for each $x \in A$ in the relation m, and binary relation s as follows:

$$xs\ y \Leftrightarrow \left[L(x) \supseteq L(y) \wedge D(x) \subseteq D(y) \wedge (\left[L(x) \supset L(y)\right] \vee \left[D(x) \subset D(y)\right])\right].$$

Then undominated alternatives on s are chosen, i.e.,

$$x \in C(\vec{P}) \Leftrightarrow \left[\,\overline{\exists}\, y \in A \,\middle|\, y\ s\ x\right].$$

Rules, using value function

18. Copeland's rule 1.

Construct function $u(x)$, which is equal to the difference of cardinalities of lower and upper contour sets of alternative x in majority relation m, i.e., $u(x) = card\big(L(x)\big) - card\big(D(x)\big)$. Then the social choice is defined by maximization of u, that is

$$x \in C(\vec{P}) \Leftrightarrow \left[\forall y \in A,\ u(x) \geq u(y)\right].$$

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[3] Note that g is a strict partial order, i.e., irreflexive and transitive binary relation.

19. Copeland's rule 2.

Function $u(x)$ is defined by cardinality of lower contour set of alternative x in majority relation m. Social choice is defined by maximization of u.

20. Copeland's rule 3.

Function $u(x)$ is constructed by cardinality of upper contour set of alternative x in majority relation m. Social choice is defined by minimization of u.

21. Young's Procedure.

If in the profile $\vec{P}$ there exists Condorcet Winner, then it is chosen. If there is no such an alternative, then all non-empty coalitions w are considered, and partial Condorcet winner $CW(w)$ is defined. The function $u(x)$ is constructed as cardinality of maximal coalition in which x is the Condorcet Winner. Then the alternative with maximal value $u(x)$ is chosen.

Rules, using tournament matrix

22. Simpson's Procedure (Maxmin Procedure).

Construct matrix S^+, such that

$$\forall a,b \in X,\ S^+ = \left(n(a,b)\right)$$

$$n(a,b) = card\left\{i \in N \ \middle|\ aP_i b\right\}, \quad n(a,a) = +\infty$$

Social choice is defined as

$$x \in C(\vec{P}) \Leftrightarrow x = \arg\max_{a \in A} \min_{b \in A}(n(a,b)) .$$

23. Minmax Procedure.

Construct matrix S^-, such that

$$\forall a,b \in X,\ S^- = \left(n(a,b)\right), \quad n(a,a) = -\infty$$

Social choice is defined as

$$x \in C(\vec{P}) \Leftrightarrow x = \arg\min_{b \in A} \max_{a \in A}(n(b,a)) .$$

Note that for the profile consisting of linear orders these rules defines the same social choice. So we give these rules both only for the sake of completeness.

q-Paretian Rules

The class of the rules considered below was introduced in Aleskerov(1985) and studied in Aleskerov(1992).

24. Strong q-Paretian simple majority rule.

Let $f(\vec{P};i,q)=\left\{x \in A \,\middle|\, card\left(D_i(x)\right) \le q\right\}$. Let $\Im=\left\{I \subset N \,\middle|\, card(I)=\left\lceil \frac{n}{2} \right\rceil\right\}$ be the family of simple majority coalitions.

Define a function

$$C(A)=\bigcup_{I\in\Im}\bigcap_{i\in I} f(\vec{P};i,q).$$

Then the social choice is defined as an alternative which is in between top alternatives for each voter in at least one simple majority coalition.If there is no such an alternative, then the social choice is defined by increasing q by 1 until it is not empty. Again, if more than one alternative are chosen, the final choice is defined by alphabetical order.

25. Strong q-Paretian plurality rule.

This rule is a counterpart of the rule 24 with the following addition. If several options are chosen, then for each alternative is counted out how many coalitions choose this alternative. Then the alternative with maximal value of this index is chosen. The rule is studied in Sertel and Kalaycioglu(1995).

26. Strongest q-Paretian simple majority rule.

Define a function

$$C(X)=\bigcap_{I\in\Im} f(\vec{P};I,q),\ \ f(\vec{P};I,q)=\{x\in A \,|\, card\left(\bigcap_{i\in I} D_i(x)\right)\le q\},\ \ card(I)=\left\lceil\frac{n}{2}\right\rceil.$$

Then the social choice is the alternative which is Pareto optimal in each simple majority coalition with $q=0$. If there is no such an alternatives, then $q=1$, $q=2$, etc., is considered until the social choice will not be empty.

Computation scheme

With small values of parameters m and n the exhaustive study of manipulability of the rules has been made. This was done via analysis of all profiles and checking of all possible orderings for all agents. For each voter $(m!-1)$ of orderings are checked. The complexity is evaluated as $(m!-1)\cdot n\cdot(m!)^n\cdot S$, where S is a complexity of choice algorithm. With $m=5$ and $n=5$ the approximate lower bound of complexity is 10^{15} which makes that exhaustive study practically impossible.[4] Hence we use the statistical scheme (Smith(forthcoming)). If we introduce random variable $n(\vec{P})$ which shows whether the profile $\vec{P}$ is manipulable in Kelly's sense, then $n(\vec{P})$ has binomial distribution with variance

$$s^2=N\cdot p\cdot(1-p).$$

[4] For some rules we made such an exhaustive study. For example, for Coombs' rule the calculations for the case $m=n=5$ took two months of work of PC-133.

Maximal value of s^2 is equal to

$$s^2 = N\left(\frac{1}{2}\right)^2 = 0{,}25 \cdot N$$

N is the number of profiles. Binomial distribution is approximated by normal distribution and 95% confidence interval is given as

$$\left(x - \frac{1{,}96 \cdot s}{\sqrt{N}},\quad x + \frac{1{,}96 \cdot s}{\sqrt{N}}\right).$$

Choosing 0.02 confidence interval we obtain the evaluation of the number of profiles

$$N \le \left[\frac{2 \cdot 1{,}96 \cdot 0{,}25}{0{,}002}\right]^2 \approx 240000\,.$$

This number has been used for all indices.

Results of experiments

The complete results of the experiments are given in the Tables 1-5 for indices K, J, I_1, I_2, I_3, respectively.

The indices K, J, I_1, I_2, and I_3 have been evaluated for all combinations of values $m = 3{,}4{,}5$, and $n = 2 \div 10$ for all rules but 21,24-26. The indices K, I_1, I_2, I_3 have been evaluated for the case $m = 5$ and $n = 51$ for all rules but 21,24-26. The indices K, J, I_1, I_2, and I_3 for the rules 21,24-26 have been evaluated for all combinations of values $m = 3{,}4{,}5$, and $n = 2 \div 5$. Already for the case $m = 5$ and $n = 4$ with few exceptions the evaluations have been made using the statistical scheme. These exceptions are as follows. For the case $m = 5, n = 4$ for the rules 1,5,6, and for the case $m = 5, n = 5$ for the rules 1,5,6,10 the exhaustive calculations were made. In these cases in Tables 1-5 one can see in the corresponding cells two values. The first one corresponds to the exhaustive evaluation, the second one was obtained using statistical scheme. Below we give some observations derived from those Tables. If one compares the rules in each group a)-e), and chooses the rule which has minimal value of the corresponding index on each pair (m, n) with $m = 3{,}5$ and $n = 3{,}5{,}7{,}9$, i.e., with odd number of alternatives and agents, then the following Table 6 can be obtained.

Table 6. Indices of Manipulability

Choice Rules	K	I_1	I_2	I_3
Scoring Rules	4	7	2, with $q = 2$	2, with $q = 2$
by Majority Relation	14-17	14-17	14-17	14-17
by Value Function	18-20	18-20	18-20	18-20
by Tournament Matrix	22,23	22,23	22,23	22,23
q-Paretian Rules	26	26	26	26

In the cells of Table 6 the numbers of rules are given.

It is immediately seen that the difference between rules is obtained on the rules from the first group, namely, on the scoring rules. The Hare rule is less manipulable in terms of Kelly's index, but if the index I_1 is considered then the less manipulable rule is the Black's one. The approval voting rule with $q = 2$ is the least manipulable rule when indices I_2 and I_3 are considered. In other words, in terms of benefit of manipulation, the approval voting rule gives less benefit (in average) to the agent.

To compare the rules in terms of degree and efficiency of manipulation Figures 1-4 are given. On these Figures the values of indices K, J, I_1, I_2, and I_3 are given, respectively. In each Figure one can see 6 graphs for the plurality, Borda's, Hare's, Black's, Fishburn's and approval voting with $q = 2$ rules constructed for the case $m = 5$ and $n = 3,5,7,9,51$. Since the corresponding values for Simpson's rule and Copeland's rule are very close to those of Fishburn's rule, those rules from groups c) and d) are not shown at the Figures.[5]

Note now that the Hare's rule for Kelly's index has the least value, a bit greater value has Fishburn's procedure. On the other hand, both plurality rule and Borda's rule has much higher values. For the index I_1 the plurality rule dominate all other rules, the least value is observed for Black's rule.

Now evaluating the efficiency of manipulation by indices I_2 and I_3, one can see that the least efficient is the approval voting rule with $q = 2$. For the case of I_2 the values for the Fishburn's rule is very close to those of the approval voting rule, and for the case of I_3 it is seen that the approval voting rule provides the least benefit (in average) of manipulation. This fact provides one more reason for a practical application of this rule. Note that both well known rules, the plurality one and the Borda's rule, are very highly manipulable with respect to both of these indices.

Since we can separate now the rules with respect to their degree and efficiency of manipulation, the results presented can be used in real situations of decision making. At the same time this study opens several problems. The first one, naturally, is how to obtain theoretical estimations of the above introduced indices which are not restricted by small values of m and n. Such estimations will provide the basis of using the different social choice rules in different practical situations.

The next question is 'Can we construct other indices to find different properties of rules under study which deal with degree and efficiency of their manipulation?' Finally, in the study above we choose the alphabetical rule of tie-breaking. However, it might be more realistic to make the analogous study considering non single-valued choice, and using generalized concept of manipulability (see, e.g., Pattanaik(1976)).

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[5] For the case $m = 5$ and $n = 51$ the total number of profiles is equal approximately to 10^{100}, from which we check only 240,000 profiles.

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Figure 1.

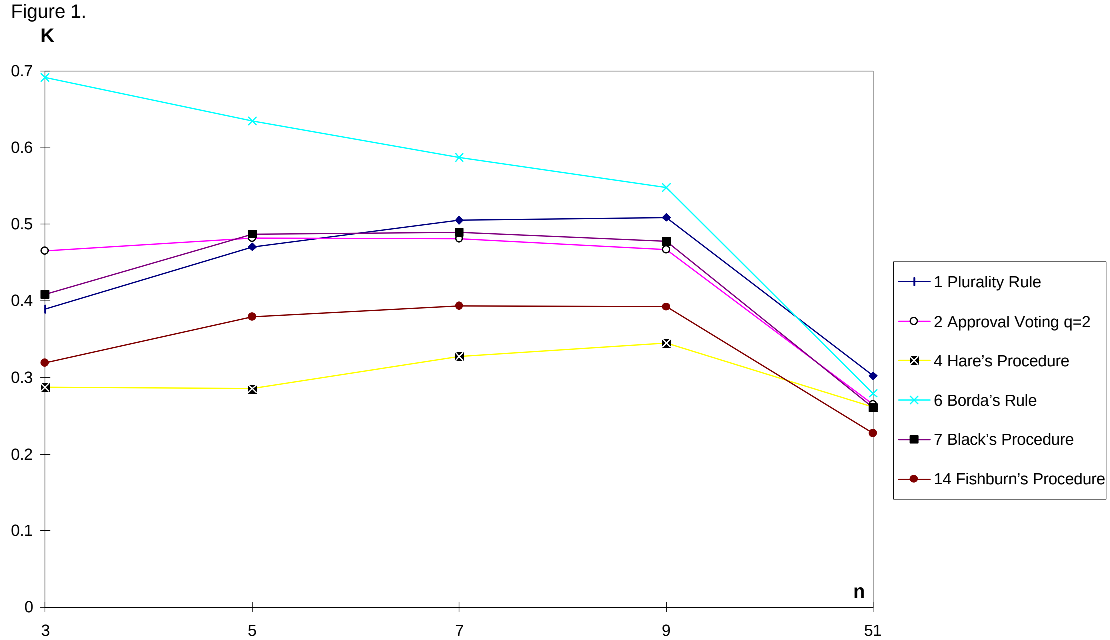

Figure 2.

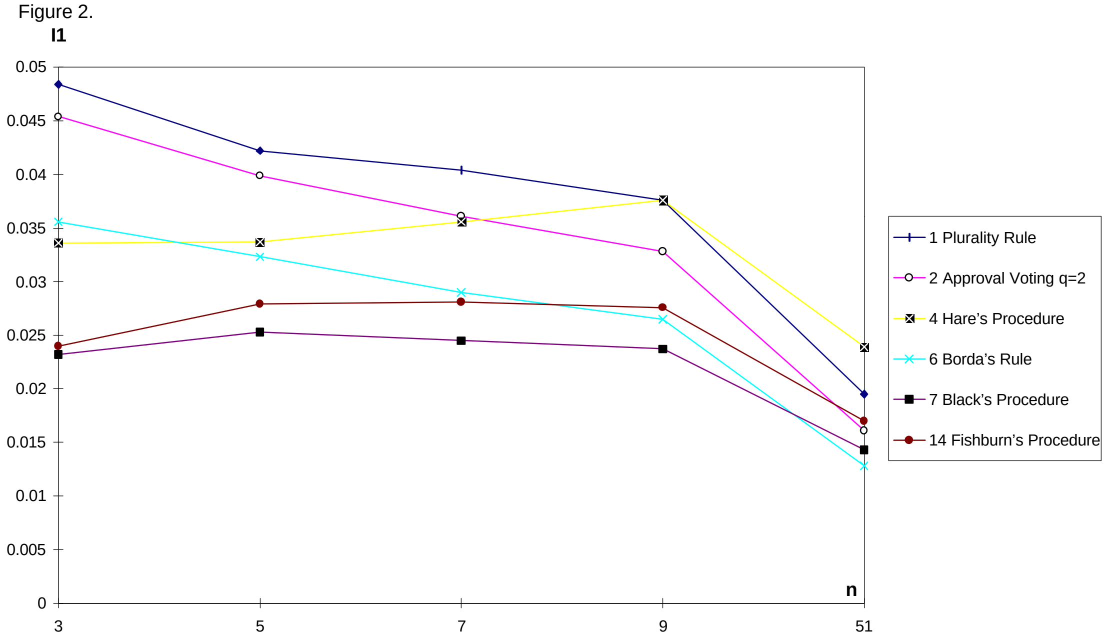

Figure 3.

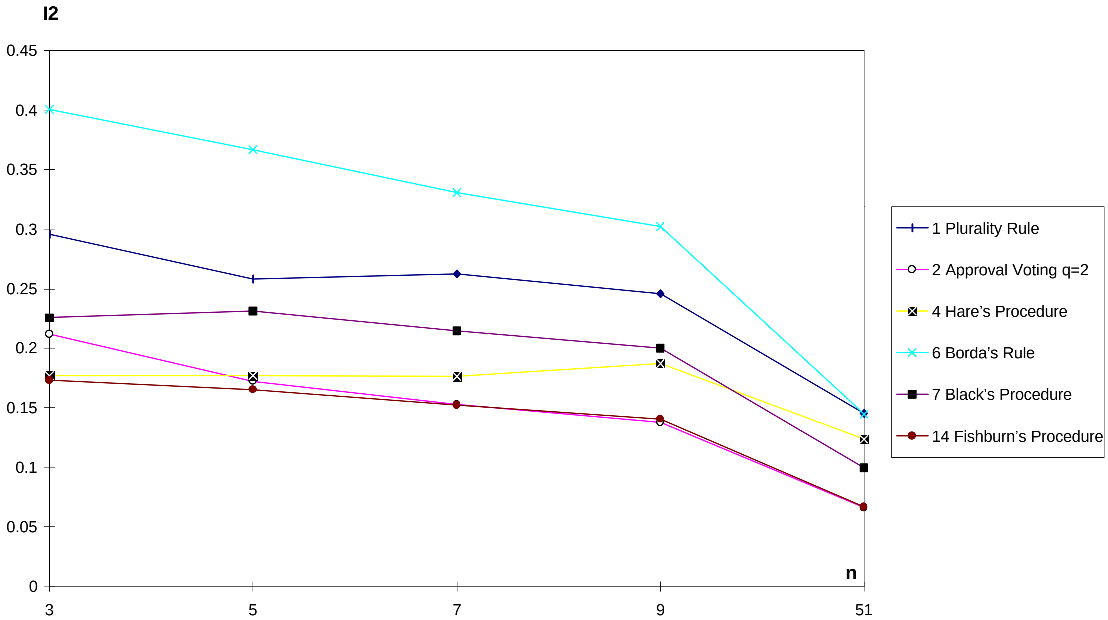

Figure 4.

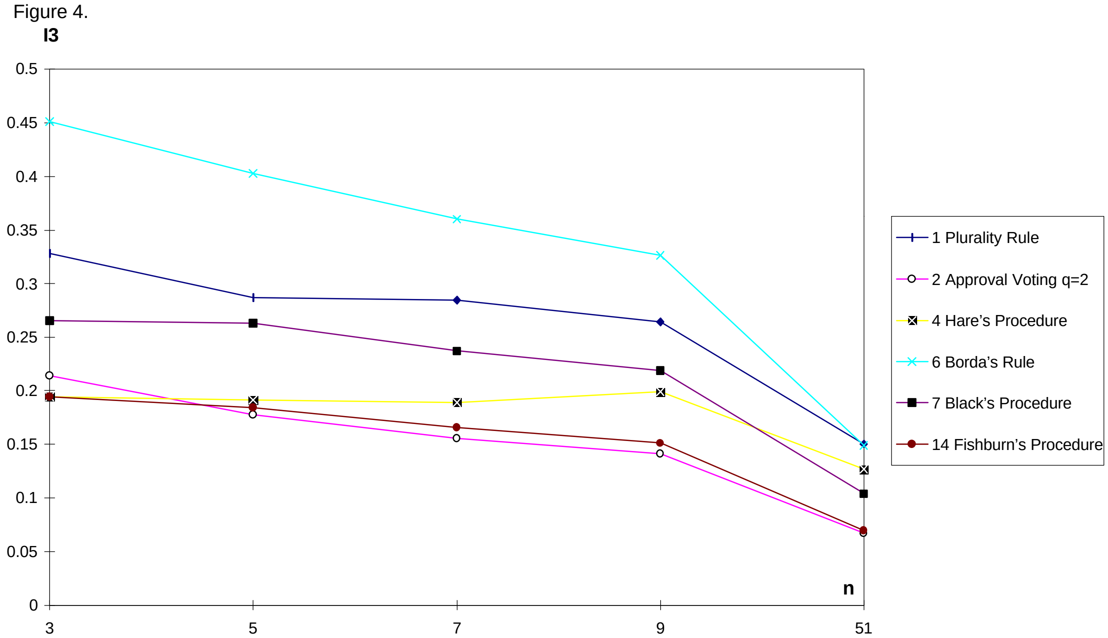

Table 1. Kelly's Index of Manipulation for Social Choice Rules

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.1111	0.1667	0.1852	0.2315	0.2393	0.2573	0.2739	0.2744	0.2855
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.1111	0.1111	0.1852	0.0926	0.1706	0.0513	0.2309	0.1193	0.2201
4	Hare's Procedure	0.1111	0.1111	0.0926	0.0926	0.0966	0.1119	0.1223	0.1207	0.1027
5	Inverse Plurality Rule	0.2500	0.2639	0.2755	0.2859	0.2878	0.2876	0.2843	0.2808	0.2787
6	Borda's Rule	0.3889	0.2361	0.3102	0.2855	0.2782	0.2705	0.2599	0.2498	0.2406
7	Black's Procedure	0.1111	0.1111	0.1435	0.1698	0.1497	0.1829	0.1511	0.1808	0.1466
8	Inverse Borda Proc.	0.1111	0.1111	0.1435	0.1389	0.1490	0.1351	0.1427	0.1298	0.1345
9	Nanson's Procedure	0.1111	0.1111	0.1435	0.1389	0.1454	0.1365	0.1405	0.1286	0.1319
10	Coombs' Procedure	0.0556	0.2222	0.2222	0.2546	0.2406	0.2577	0.2379	0.2552	0.2482
11	Dominant Set	0.1667	0.1111	0.2361	0.1389	0.2668	0.1458	0.2790	0.1435	0.2799
12	Undominated Set	0.1111	0.1111	0.1435	0.1389	0.1531	0.1453	0.1524	0.1431	0.1481
13	Min. Weakly Stable Set	0.1111	0.1111	0.1435	0.1389	0.1515	0.1458	0.1520	0.1436	0.1465
14	Fishburn's Procedure	0.1111	0.1111	0.1435	0.1389	0.1487	0.1450	0.1455	0.1430	0.1385
15	Uncovered Set I	0.3889	0.1111	0.3704	0.1389	0.3437	0.1431	0.3173	0.1433	0.2969
16	Uncovered Set II	0.1111	0.1111	0.1481	0.1389	0.1601	0.1454	0.1579	0.1441	0.1552
17	Richelson's Procedure	0.3889	0.1111	0.3704	0.1389	0.3431	0.1458	0.3187	0.1430	0.2966
18	Copeland's Rule I	0.3889	0.1111	0.3704	0.1389	0.3439	0.1453	0.3181	0.1443	0.2956
19	Copeland's Rule II	0.1111	0.1111	0.1435	0.1389	0.1502	0.1459	0.1465	0.1442	0.1395
20	Copeland's Rule III	0.3889	0.1111	0.2963	0.1389	0.2453	0.1461	0.2148	0.1429	0.1942
21	Young's Procedure	0.1111	0.1111	0.1435	0.1389	-	-	-	-	-
22	Maxmin	0.1111	0.1111	0.1435	0.1389	0.1501	0.1427	0.1472	0.1409	0.1391
23	Minmax	0.1111	0.1111	0.1435	0.1389	0.1496	0.1429	0.1454	0.1400	0.1399
24	Strong q-Par. sim. maj.	0.1667	0.1389	0.2130	0.2160	-	-	-	-	-
25	Strong q-Par. plurality	0.1667	0.1111	0.2269	0.1736	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.1111	0.1111	0.1435	0.1389	-	-	-	-	-

Table 1. Kelly's Index of Manipulation for Social Choice Rules

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.2083	0.2943	0.3247	0.3738	0.3981	0.4055	0.4133	0.4199	0.4195
2a	Approval Voting, q=2	0.3750	0.3941	0.4256	0.4326	0.4359	0.4338	0.4302	0.4251	0.4196
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.2083	0.2083	0.3212	0.2042	0.3527	0.2474	0.3322	0.2389	0.3175
4	Hare's Procedure	0.2083	0.2083	0.1888	0.1961	0.2045	0.2263	0.2448	0.2392	0.2122
5	Inverse Plurality Rule	0.4063	0.5000	0.5254	0.5238	0.5206	0.5174	0.5149	0.5025	0.4884
6	Borda's Rule	0.6042	0.5122	0.5002	0.5044	0.4790	0.4643	0.4485	0.4315	0.4178
7	Black's Procedure	0.2083	0.2760	0.2635	0.3529	0.2764	0.3622	0.2796	0.3537	0.2748
8	Inverse Borda Proc.	0.2083	0.2218	0.2662	0.2678	0.2754	0.2612	0.2659	0.2483	0.2515
9	Nanson's Procedure	0.2083	0.2274	0.2633	0.2657	0.2702	0.2585	0.2611	0.2503	0.2476
10	Coombs' Procedure	0.1146	0.3967	0.4367	0.4601	0.4417	0.4702	0.4474	0.4576	0.4473
11	Dominant Set	0.2465	0.2257	0.3384	0.2986	0.3722	0.3242	0.3804	0.3290	0.3787
12	Undominated Set	0.2083	0.2257	0.2635	0.2986	0.2750	0.3243	0.2742	0.3289	0.2682
13	Min. Weakly Stable Set	0.2083	0.2266	0.2635	0.2852	0.2741	0.2998	0.2718	0.3008	0.2616
14	Fishburn's Procedure	0.2083	0.2222	0.2635	0.2699	0.2742	0.2808	0.2711	0.2784	0.2623
15	Uncovered Set I	0.6042	0.2222	0.5525	0.2699	0.5262	0.2815	0.5035	0.2776	0.4798
16	Uncovered Set II	0.2083	0.2222	0.2749	0.2699	0.2939	0.2806	0.2964	0.2782	0.2875
17	Richelson's Procedure	0.6042	0.2222	0.5577	0.2699	0.5293	0.2804	0.5058	0.2796	0.4816
18	Copeland's Rule I	0.6042	0.2274	0.5501	0.2794	0.5281	0.2933	0.5062	0.2923	0.4842
19	Copeland's Rule II	0.2083	0.2274	0.2635	0.2794	0.2756	0.2923	0.2717	0.2923	0.2626
20	Copeland's Rule III	0.6042	0.2274	0.4533	0.2794	0.3970	0.2926	0.3663	0.2922	0.3418
21	Young's Procedure	0.2083	0.2378	0.2635	0.2897	-	-	-	-	-
22	Maxmin	0.2083	0.2378	0.2635	0.2897	0.2768	0.2932	0.2725	0.2838	0.2642
23	Minmax	0.2083	0.2378	0.2635	0.2897	0.2766	0.2928	0.2720	0.2831	0.2647
24	Strong q-Par. sim. maj.	0.3472	0.2257	0.3828	0.3445	-	-	-	-	-
25	Strong q-Par. plurality	0.3472	0.2257	0.3898	0.3522	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.2083	0.2179	0.2635	0.2787	-	-	-	-	-

Table 1. Kelly's Index of Manipulation for Social Choice Rules

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8
1	Plurality Rule	0.2867	0.3890	0.4257/0.4262	0.4707/0.4717	0.4951	0.5051	0.5063
2a	Approval Voting, q=2	0.4300	0.4650	0.4781	0.4821	0.4864	0.4811	0.4779
2b	Approval Voting, q=3	0.5389	0.5758	0.5813	0.5899	0.5803	0.5706	0.5579
3	Run-Off Procedure	0.2867	0.2870	0.4197	0.3003	0.4317	0.3573	0.4147
4	Hare's Procedure	0.2867	0.2870	0.2728	0.2852	0.3010	0.3278	0.3487
5	Inverse Plurality Rule	0.4900	0.6110	0.6875/0.6882	0.7148/0.7164	0.7106	0.6937	0.6766
6	Borda's Rule	0.7100	0.6914	0.6390/0.6379	0.6347/0.6363	0.6098	0.5877	0.5700
7	Black's Procedure	0.2867	0.4086	0.3572	0.4873	0.3776	0.4894	0.3807
8	Inverse Borda Proc.	0.2867	0.3182	0.3623	0.3743	0.3732	0.3632	0.3627
9	Nanson's Procedure	0.2867	0.3423	0.3600	0.3937	0.3677	0.3817	0.3573
10	Coombs' Procedure	0.1683	0.5177	0.5925	0.6212/0.6203	0.5997	0.6273	0.5999
11	Dominant Set	0.3161	0.3249	0.4253	0.4092	0.4664	0.4307	0.4785
12	Undominated Set	0.2867	0.3249	0.3577	0.4082	0.3712	0.4310	0.3672
13	Min. Weakly Stable Set	0.2867	0.3269	0.3557	0.4096	0.3719	0.4296	0.3701
14	Fishburn's Procedure	0.2867	0.3189	0.3561	0.3795	0.3728	0.3936	0.3661
15	Uncovered Set I	0.7100	0.3189	0.6630	0.3800	0.6441	0.3944	0.6221
16	Uncovered Set II	0.2867	0.3189	0.3752	0.3793	0.3947	0.3926	0.3983
17	Richelson's Procedure	0.7100	0.3189	0.6672	0.3808	0.6491	0.3940	0.6248
18	Copeland's Rule I	0.7100	0.3287	0.6531	0.3951	0.6351	0.4123	0.6150
19	Copeland's Rule II	0.2867	0.3287	0.3556	0.3951	0.3728	0.4124	0.3700
20	Copeland's Rule III	0.7100	0.3287	0.5606	0.3952	0.5114	0.4117	0.4763
21	Young's Procedure	0.2867	0.3440	0.3558	0.4072	-	-	-
22	Maxmin	0.2867	0.3440	0.3555	0.4059	0.3741	0.4039	0.3715
23	Minmax	0.2867	0.3440	0.3547	0.4047	0.3737	0.4039	0.3705
24	Strong q-Par. sim. maj.	0.3667	0.3000	0.4569	0.4397	-	-	-
25	Strong q-Par. plurality	0.3667	0.3000	0.4682	0.4365	-	-	-
26	Strongest q-Par. sim. maj	0.2867	0.3063	0.3549	0.3886	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_1 (share of profiles in which exactly one voter manipulates)

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.1111	0.1111	0.1481	0.1698	0.1546	0.1675	0.1629	0.1513	0.1462
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.1111	0.1111	0.1481	0.0000	0.1687	0.0072	0.1592	0.0037	0.1111
4	Hare's Procedure	0.1111	0.1111	0.0370	0.0000	0.0000	0.0070	0.0040	0.0042	0.0019
5	Inverse Plurality Rule	0.1667	0.1528	0.1389	0.1100	0.0976	0.0819	0.0630	0.0540	0.0452
6	Borda's Rule	0.3889	0.1944	0.1296	0.1427	0.0642	0.0622	0.0439	0.0286	0.0242
7	Black's Procedure	0.1111	0.1111	0.1111	0.1235	0.0903	0.1020	0.0703	0.0759	0.0516
8	Inverse Borda Proc.	0.1111	0.1111	0.1111	0.0617	0.0744	0.0446	0.0489	0.0311	0.0306
9	Nanson's Procedure	0.1111	0.1111	0.1111	0.0617	0.0816	0.0459	0.0583	0.0311	0.0383
10	Coombs' Procedure	0.0556	0.1667	0.1667	0.2122	0.2061	0.2184	0.1718	0.1598	0.1332
11	Dominant Set	0.1111	0.1111	0.1481	0.1080	0.1432	0.0868	0.1227	0.0638	0.0968
12	Undominated Set	0.1111	0.1111	0.1111	0.1080	0.0978	0.0864	0.0750	0.0629	0.0546
13	Min. Weakly Stable Set	0.1111	0.1111	0.1111	0.1080	0.0960	0.0866	0.0752	0.0627	0.0530
14	Fishburn's Procedure	0.1111	0.1111	0.1111	0.1080	0.0898	0.0863	0.0657	0.0640	0.0456
15	Uncovered Set I	0.3889	0.1111	0.2222	0.1080	0.1559	0.0847	0.1065	0.0631	0.0720
16	Uncovered Set II	0.1111	0.1111	0.0926	0.1080	0.0761	0.0865	0.0613	0.0633	0.0488
17	Richelson's Procedure	0.3889	0.1111	0.2222	0.1080	0.1561	0.0867	0.1063	0.0631	0.0726
18	Copeland's Rule I	0.3889	0.1111	0.2222	0.1080	0.1555	0.0864	0.1066	0.0630	0.0723
19	Copeland's Rule II	0.1111	0.1111	0.1111	0.1080	0.0909	0.0862	0.0664	0.0642	0.0459
20	Copeland's Rule III	0.3889	0.1111	0.1852	0.1080	0.1075	0.0867	0.0714	0.0633	0.0487
21	Young's Procedure	0.1111	0.1111	0.1111	0.1042	-	-	-	-	-
22	Maxmin	0.1111	0.1111	0.1111	0.1042	0.0899	0.0808	0.0671	0.0581	0.0465
23	Minmax	0.1111	0.1111	0.1111	0.1042	0.0898	0.0808	0.0665	0.0575	0.0469
24	Strong q-Par. sim. maj.	0.1667	0.0833	0.1667	0.1350	-	-	-	-	-
25	Strong q-Par. plurality	0.1667	0.1111	0.1389	0.1157	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.1111	0.1111	0.1111	0.1080	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_1 (share of profiles in which exactly one voter manipulates)

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.2083	0.1823	0.2083	0.2230	0.1999	0.1846	0.1673	0.1470	0.1266
2a	Approval Voting, q=2	0.3333	0.3108	0.2836	0.2590	0.2244	0.1964	0.1687	0.1470	0.1261
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.2083	0.1823	0.2188	0.0101	0.2050	0.0242	0.1175	0.0223	0.0538
4	Hare's Procedure	0.2083	0.1823	0.0672	0.0168	0.0185	0.0264	0.0210	0.0171	0.0104
5	Inverse Plurality Rule	0.3125	0.2813	0.2422	0.2002	0.1637	0.1273	0.1080	0.0871	0.0697
6	Borda's Rule	0.5799	0.3902	0.1628	0.1731	0.1103	0.0816	0.0638	0.0474	0.0369
7	Black's Procedure	0.1979	0.2270	0.1846	0.2112	0.1456	0.1698	0.1134	0.1256	0.0847
8	Inverse Borda Proc.	0.1979	0.1979	0.1787	0.1091	0.1133	0.0763	0.0736	0.0528	0.0469
9	Nanson's Procedure	0.1979	0.2101	0.1845	0.1289	0.1267	0.0952	0.0853	0.0717	0.0568
10	Coombs' Procedure	0.1111	0.2708	0.2768	0.2972	0.2704	0.2710	0.2266	0.1959	0.1691
11	Dominant Set	0.1181	0.2005	0.2021	0.2114	0.1844	0.1847	0.1503	0.1492	0.1178
12	Undominated Set	0.1979	0.2005	0.1846	0.2114	0.1452	0.1840	0.1094	0.1490	0.0816
13	Min. Weakly Stable Set	0.1979	0.2066	0.1846	0.2006	0.1455	0.1635	0.1083	0.1260	0.0762
14	Fishburn's Procedure	0.1979	0.2023	0.1846	0.1823	0.1431	0.1451	0.1065	0.1091	0.0763
15	Uncovered Set I	0.5799	0.2023	0.2382	0.1823	0.1827	0.1458	0.1403	0.1098	0.1046
16	Uncovered Set II	0.1979	0.2023	0.1462	0.1823	0.1202	0.1451	0.1000	0.1095	0.0796
17	Richelson's Procedure	0.5799	0.2023	0.2380	0.1823	0.1781	0.1460	0.1367	0.1105	0.1007
18	Copeland's Rule I	0.5799	0.1992	0.2454	0.1866	0.1893	0.1540	0.1501	0.1216	0.1166
19	Copeland's Rule II	0.1979	0.1992	0.1846	0.1866	0.1448	0.1548	0.1064	0.1210	0.0769
20	Copeland's Rule III	0.5799	0.1992	0.2057	0.1866	0.1354	0.1543	0.1069	0.1204	0.0854
21	Young's Procedure	0.1979	0.2196	0.1846	0.1966	-	-	-	-	-
22	Maxmin	0.1979	0.2188	0.1846	0.1966	0.1471	0.1472	0.1091	0.1046	0.0785
23	Minmax	0.1979	0.2188	0.1846	0.1966	0.1460	0.1463	0.1082	0.1046	0.0790
24	Strong q-Par. sim. maj.	0.3333	0.1181	0.2720	0.1911	-	-	-	-	-
25	Strong q-Par. plurality	0.3333	0.1597	0.2765	0.2043	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.1979	0.1927	0.1846	0.1932	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_1 (share of profiles in which exactly one voter manipulates)

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8
1	Plurality Rule	0.2867	0.2260	0.2315/0.2316	0.2351/0.2356	0.2034	0.1753	0.1508
2a	Approval Voting, q=2	0.3900	0.3625	0.3073	0.2638	0.2274	0.1931	0.1645
2b	Approval Voting, q=3	0.3444	0.3371	0.2879	0.2349	0.1939	0.1586	0.1254
3	Run-Off Procedure	0.2867	0.2260	0.2519	0.0215	0.1899	0.0491	0.0782
4	Hare’s Procedure	0.2867	0.2260	0.0879	0.0338	0.0375	0.0447	0.0365
5	Inverse Plurality Rule	0.3800	0.3630	0.3060/0.3077	0.2519/0.2520	0.1986	0.1545	0.1206
6	Borda’s Rule	0.6281	0.4740	0.1785/0.1790	0.1562/0.1560	0.1164	0.0840	0.0678
7	Black’s Procedure	0.2621	0.2924	0.2295	0.2422	0.1766	0.1860	0.1370
8	Inverse Borda Proc.	0.2621	0.2522	0.2145	0.1402	0.1295	0.0966	0.0840
9	Nanson’s Procedure	0.2621	0.2754	0.2267	0.1668	0.1474	0.1198	0.0984
10	Coombs’ Procedure	0.1590	0.3257	0.3095	0.3108/0.3102	0.2744	0.2565	0.2201
11	Dominant Set	0.1329	0.2707	0.2396	0.2581	0.2094	0.2101	0.1693
12	Undominated Set	0.2621	0.2707	0.2288	0.2573	0.1756	0.2106	0.1294
13	Min. Weakly Stable Set	0.2621	0.2715	0.2268	0.2557	0.1765	0.2070	0.1310
14	Fishburn’s Procedure	0.2621	0.2657	0.2276	0.2259	0.1744	0.1804	0.1290
15	Uncovered Set I	0.6281	0.2657	0.2267	0.2258	0.1836	0.1787	0.1479
16	Uncovered Set II	0.2621	0.2657	0.1741	0.2262	0.1422	0.1793	0.1183
17	Richelson’s Procedure	0.6281	0.2657	0.2202	0.2269	0.1767	0.1800	0.1399
18	Copeland’s Rule I	0.6281	0.2596	0.2351	0.2298	0.1885	0.1915	0.1545
19	Copeland’s Rule II	0.2621	0.2596	0.2277	0.2310	0.1754	0.1927	0.1298
20	Copeland’s Rule III	0.6281	0.2596	0.2041	0.2303	0.1483	0.1923	0.1237
21	Young’s Procedure	0.2621	0.2960	0.2286	0.2465	-	-	-
22	Maxmin	0.2621	0.2939	0.2274	0.2462	0.1768	0.1757	0.1330
23	Minmax	0.2621	0.2939	0.2279	0.2446	0.1773	0.1772	0.1333
24	Strong q-Par. sim. maj.	0.3500	0.1575	0.2927	0.2235	-	-	-
25	Strong q-Par. plurality	0.3500	0.1800	0.2955	0.2100	-	-	-
26	Strongest q-Par. sim. maj	0.2621	0.2495	0.2281	0.2404	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_2 (share of profiles in which exactly two voters manipulate)

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.0000	0.0556	0.0370	0.0463	0.0462	0.0596	0.0693	0.0678	0.0834
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0000	0.0000	0.0370	0.0926	0.0466	0.0798	0.0239	0.0381	0.0594
4	Hare’s Procedure	0.0000	0.0000	0.0556	0.0926	0.0863	0.0787	0.0571	0.0385	0.0165
5	Inverse Plurality Rule	0.0833	0.0972	0.1065	0.1196	0.1167	0.1116	0.1031	0.0949	0.0853
6	Borda’s Rule	0.0000	0.0417	0.1806	0.0926	0.1234	0.1076	0.0654	0.0673	0.0447
7	Black’s Procedure	0.0000	0.0000	0.0324	0.0386	0.0462	0.0546	0.0504	0.0629	0.0519
8	Inverse Borda Proc.	0.0000	0.0000	0.0324	0.0772	0.0589	0.0317	0.0420	0.0283	0.0375
9	Nanson’s Procedure	0.0000	0.0000	0.0324	0.0772	0.0506	0.0315	0.0444	0.0298	0.0431
10	Coombs’ Procedure	0.0000	0.0556	0.0556	0.0270	0.0151	0.0127	0.0354	0.0476	0.0721
11	Dominant Set	0.0556	0.0000	0.0463	0.0309	0.0627	0.0511	0.0814	0.0582	0.0905
12	Undominated Set	0.0000	0.0000	0.0324	0.0309	0.0421	0.0509	0.0527	0.0581	0.0580
13	Min. Weakly Stable Set	0.0000	0.0000	0.0324	0.0309	0.0426	0.0515	0.0530	0.0590	0.0573
14	Fishburn’s Procedure	0.0000	0.0000	0.0324	0.0309	0.0459	0.0507	0.0517	0.0577	0.0514
15	Uncovered Set I	0.0000	0.0000	0.1204	0.0309	0.1019	0.0506	0.1049	0.0584	0.0951
16	Uncovered Set II	0.0000	0.0000	0.0556	0.0309	0.0528	0.0509	0.0412	0.0588	0.0382
17	Richelson’s Procedure	0.0000	0.0000	0.1204	0.0309	0.1020	0.0512	0.1037	0.0587	0.0955
18	Copeland’s Rule I	0.0000	0.0000	0.1204	0.0309	0.1023	0.0512	0.1039	0.0595	0.0949
19	Copeland’s Rule II	0.0000	0.0000	0.0324	0.0309	0.0461	0.0516	0.0518	0.0586	0.0518
20	Copeland’s Rule III	0.0000	0.0000	0.1111	0.0309	0.0996	0.0512	0.0802	0.0580	0.0683
21	Young’s Procedure	0.0000	0.0000	0.0324	0.0347	-	-	-	-	-
22	Maxmin	0.0000	0.0000	0.0324	0.0347	0.0466	0.0485	0.0509	0.0542	0.0513
23	Minmax	0.0000	0.0000	0.0324	0.0347	0.0465	0.0484	0.0501	0.0540	0.0518
24	Strong q-Par. sim. maj.	0.0000	0.0556	0.0000	0.0424	-	-	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0694	0.0116	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0000	0.0324	0.0309	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_2 (share of profiles in which exactly two voters manipulate)

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.0000	0.1120	0.0885	0.1090	0.1122	0.1101	0.1244	0.1264	0.1248
2a	Approval Voting, q=2	0.0417	0.0747	0.1233	0.1385	0.1520	0.1578	0.1545	0.1518	0.1430
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0000	0.0260	0.0885	0.1748	0.0798	0.1209	0.0997	0.0395	0.1246
4	Hare's Procedure	0.0000	0.0260	0.1169	0.1702	0.1473	0.1259	0.0960	0.0671	0.0378
5	Inverse Plurality Rule	0.0938	0.1875	0.1992	0.1941	0.1861	0.1759	0.1583	0.1396	0.1209
6	Borda's Rule	0.0243	0.1220	0.3156	0.2052	0.1540	0.1409	0.1026	0.0842	0.0699
7	Black's Procedure	0.0104	0.0490	0.0742	0.0868	0.0921	0.0914	0.0902	0.1007	0.0881
8	Inverse Borda Proc.	0.0104	0.0234	0.0817	0.1390	0.1063	0.0590	0.0725	0.0495	0.0601
9	Nanson's Procedure	0.0104	0.0174	0.0741	0.1213	0.1008	0.0638	0.0794	0.0558	0.0682
10	Coombs' Procedure	0.0035	0.1198	0.1349	0.1056	0.0946	0.1026	0.1137	0.1259	0.1356
11	Dominant Set	0.1285	0.0182	0.0752	0.0642	0.1043	0.0956	0.1215	0.1094	0.1227
12	Undominated Set	0.0104	0.0182	0.0742	0.0642	0.0896	0.0964	0.0929	0.1105	0.0863
13	Min. Weakly Stable Set	0.0104	0.0182	0.0742	0.0694	0.0908	0.0992	0.0929	0.1092	0.0891
14	Fishburn's Procedure	0.0104	0.0200	0.0742	0.0742	0.0916	0.0946	0.0936	0.1000	0.0882
15	Uncovered Set I	0.0243	0.0200	0.2384	0.0742	0.1480	0.0953	0.1359	0.0989	0.1248
16	Uncovered Set II	0.0104	0.0200	0.1201	0.0742	0.0916	0.0943	0.0704	0.0990	0.0630
17	Richelson's Procedure	0.0243	0.0200	0.2471	0.0742	0.1464	0.0941	0.1336	0.0995	0.1215
18	Copeland's Rule I	0.0243	0.0282	0.2434	0.0742	0.1543	0.0928	0.1336	0.0967	0.1228
19	Copeland's Rule II	0.0104	0.0282	0.0742	0.0742	0.0907	0.0906	0.0927	0.0969	0.0872
20	Copeland's Rule III	0.0243	0.0282	0.2340	0.0742	0.1433	0.0916	0.1029	0.0975	0.0862
21	Young's Procedure	0.0104	0.0165	0.0742	0.0769	-	-	-	-	-
22	Maxmin	0.0104	0.0165	0.0742	0.0767	0.0902	0.0979	0.0896	0.1002	0.0867
23	Minmax	0.0104	0.0165	0.0742	0.0767	0.0905	0.0985	0.0906	0.0994	0.0867
24	Strong q-Par. sim. maj.	0.0139	0.1076	0.0605	0.1050	-	-	-	-	-
25	Strong q-Par. plurality	0.0139	0.0660	0.0638	0.0848	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0104	0.0234	0.0742	0.0670	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_2 (share of profiles in which exactly two voters manipulate)

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8
1	Plurality Rule	0.0000	0.1630	0.1325/0.1326	0.1508/0.1506	0.1522	0.1419	0.1431
2a	Approval Voting, q=2	0.0400	0.0900	0.1416	0.1619	0.1691	0.1670	0.1646
2b	Approval Voting, q=3	0.1944	0.1991	0.2151	0.2259	0.2165	0.2012	0.1823
3	Run-Off Procedure	0.0000	0.0610	0.1327	0.2235	0.1128	0.1289	0.1490
4	Hare's Procedure	0.0000	0.0610	0.1687	0.2217	0.1865	0.1555	0.1172
5	Inverse Plurality Rule	0.1100	0.2070	0.2670/0.2661	0.2611/0.2620	0.2382	0.2105	0.1821
6	Borda's Rule	0.0819	0.2146	0.3927/0.3911	0.2625/0.2619	0.1642	0.1401	0.1085
7	Black's Procedure	0.0246	0.1150	0.1145	0.1248	0.1285	0.1199	0.1169
8	Inverse Borda Proc.	0.0246	0.0632	0.1306	0.181	0.1394	0.0776	0.0905
9	Nanson's Procedure	0.0246	0.0668	0.1195	0.1674	0.1350	0.0813	0.1002
10	Coombs' Procedure	0.0093	0.1763	0.2126	0.1811/0.1810	0.1590	0.1632	0.1592
11	Dominant Set	0.0832	0.0477	0.0941	0.1083	0.1368	0.1387	0.1515
12	Undominated Set	0.0246	0.0477	0.1158	0.1081	0.1271	0.1395	0.1221
13	Min. Weakly Stable Set	0.0246	0.0496	0.1162	0.1101	0.1271	0.1387	0.1222
14	Fishburn's Procedure	0.0246	0.0519	0.115	0.1159	0.1271	0.1263	0.1197
15	Uncovered Set I	0.0819	0.0519	0.2967	0.1152	0.1654	0.1280	0.1463
16	Uncovered Set II	0.0246	0.0519	0.1764	0.1147	0.1130	0.1274	0.0875
17	Richelson's Procedure	0.0819	0.0519	0.3077	0.1154	0.1602	0.1269	0.1400
18	Copeland's Rule I	0.0819	0.0691	0.3107	0.1102	0.1741	0.1180	0.1434
19	Copeland's Rule II	0.0246	0.0691	0.1148	0.1102	0.1254	0.1167	0.1203
20	Copeland's Rule III	0.0819	0.0691	0.3139	0.1106	0.1618	0.1168	0.1126
21	Young's Procedure	0.0246	0.0434	0.1141	0.1198	-	-	-
22	Maxmin	0.0246	0.0432	0.1149	0.118	0.1256	0.1353	0.1183
23	Minmax	0.0246	0.0432	0.1137	0.1188	0.1248	0.1355	0.1166
24	Strong q-Par. sim. maj.	0.0167	0.1425	0.0994	0.146	-	-	-
25	Strong q-Par. plurality	0.0167	0.1200	0.1219	0.1481	-	-	-
26	Strongest q-Par. sim. maj	0.0246	0.0520	0.1142	0.1025	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_3 (share of profiles in which exactly three voters manipulate)

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.0000	0.0000	0.0000	0.0154	0.0307	0.0241	0.0264	0.0265	0.0317
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0000	0.0000	0.0000	0.0000	0.0154	0.0260	0.0318	0.0583	0.0239
4	Hare's Procedure	0.0000	0.0000	0.0000	0.0000	0.0102	0.0262	0.0441	0.0589	0.0631
5	Inverse Plurality Rule	0.0000	0.0139	0.0278	0.0502	0.0602	0.0697	0.0811	0.0835	0.0853
6	Borda's Rule	0.0000	0.0000	0.0000	0.0502	0.0855	0.0621	0.1003	0.0789	0.0652
7	Black's Procedure	0.0000	0.0000	0.0000	0.0077	0.0132	0.0189	0.0240	0.0248	0.0251
8	Inverse Borda Proc.	0.0000	0.0000	0.0000	0.0000	0.0157	0.0588	0.0428	0.0255	0.0262
9	Nanson's Procedure	0.0000	0.0000	0.0000	0.0000	0.0132	0.0592	0.0319	0.0231	0.0249
10	Coombs' Procedure	0.0000	0.0000	0.0000	0.0154	0.0116	0.0205	0.0163	0.0071	0.0061
11	Dominant Set	0.0000	0.0000	0.0370	0.0000	0.0296	0.0080	0.0270	0.0194	0.0373
12	Undominated Set	0.0000	0.0000	0.0000	0.0000	0.0132	0.0079	0.0188	0.0201	0.0235
13	Min. Weakly Stable Set	0.0000	0.0000	0.0000	0.0000	0.0128	0.0077	0.0181	0.0200	0.0244
14	Fishburn's Procedure	0.0000	0.0000	0.0000	0.0000	0.0129	0.0079	0.0222	0.0194	0.0271
15	Uncovered Set I	0.0000	0.0000	0.0278	0.0000	0.0590	0.0078	0.0450	0.0198	0.0563
16	Uncovered Set II	0.0000	0.0000	0.0000	0.0000	0.0312	0.0080	0.0391	0.0200	0.0305
17	Richelson's Procedure	0.0000	0.0000	0.0278	0.0000	0.0580	0.0079	0.0463	0.0192	0.0555
18	Copeland's Rule I	0.0000	0.0000	0.0278	0.0000	0.0589	0.0078	0.0458	0.0199	0.0554
19	Copeland's Rule II	0.0000	0.0000	0.0000	0.0000	0.0133	0.0081	0.0224	0.0195	0.0272
20	Copeland's Rule III	0.0000	0.0000	0.0000	0.0000	0.0382	0.0082	0.0484	0.0196	0.0452
21	Young's Procedure	0.0000	0.0000	0.0000	0.0000	-	-	-	-	-
22	Maxmin	0.0000	0.0000	0.0000	0.0000	0.0136	0.0134	0.0232	0.0223	0.0254
23	Minmax	0.0000	0.0000	0.0000	0.0000	0.0133	0.0137	0.0230	0.0219	0.0254
24	Strong q-Par. sim. maj.	0.0000	0.0000	0.0463	0.0309	-	-	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0185	0.0463	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0000	0.0000	0.0000	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_3 (share of profiles in which exactly three voters manipulate)

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.0000	0.0000	0.0278	0.0418	0.0649	0.0641	0.0655	0.0729	0.0745
2a	Approval Voting, q=2	0.0000	0.0087	0.0174	0.0305	0.0497	0.0627	0.0781	0.0868	0.0957
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0000	0.0000	0.0139	0.0183	0.0573	0.0896	0.0637	0.1207	0.0470
4	Hare’s Procedure	0.0000	0.0000	0.0047	0.0091	0.0349	0.0640	0.0866	0.1015	0.1007
5	Inverse Plurality Rule	0.0000	0.0313	0.0703	0.0952	0.1126	0.1281	0.1303	0.1279	0.1231
6	Borda’s Rule	0.0000	0.0000	0.0209	0.1229	0.1824	0.1383	0.1270	0.1112	0.0913
7	Black’s Procedure	0.0000	0.0000	0.0045	0.0521	0.0346	0.0557	0.0520	0.0475	0.0519
8	Inverse Borda Proc.	0.0000	0.0004	0.0052	0.0169	0.0496	0.1093	0.0795	0.0471	0.0486
9	Nanson’s Procedure	0.0000	0.0000	0.0045	0.0150	0.0391	0.0874	0.0689	0.0478	0.0504
10	Coombs’ Procedure	0.0000	0.0061	0.0230	0.0501	0.0461	0.0506	0.0460	0.0413	0.0466
11	Dominant Set	0.0000	0.0069	0.0494	0.0156	0.0459	0.0265	0.0516	0.0424	0.0673
12	Undominated Set	0.0000	0.0069	0.0045	0.0156	0.0360	0.0261	0.0483	0.0418	0.0547
13	Min. Weakly Stable Set	0.0000	0.0017	0.0045	0.0117	0.0344	0.0261	0.0500	0.0452	0.0547
14	Fishburn’s Procedure	0.0000	0.0000	0.0045	0.0122	0.0357	0.0320	0.0485	0.0468	0.0547
15	Uncovered Set I	0.0000	0.0000	0.0746	0.0122	0.1289	0.0313	0.0867	0.0463	0.0876
16	Uncovered Set II	0.0000	0.0000	0.0080	0.0122	0.0739	0.0318	0.0707	0.0473	0.0520
17	Richelson’s Procedure	0.0000	0.0000	0.0717	0.0122	0.1362	0.0314	0.0880	0.0471	0.0866
18	Copeland’s Rule I	0.0000	0.0000	0.0607	0.0164	0.1265	0.0347	0.0936	0.0471	0.0855
19	Copeland’s Rule II	0.0000	0.0000	0.0045	0.0164	0.0361	0.0355	0.0497	0.0479	0.0546
20	Copeland’s Rule III	0.0000	0.0000	0.0134	0.0164	0.1070	0.0346	0.0908	0.0473	0.0701
21	Young’s Procedure	0.0000	0.0017	0.0045	0.0134	-	-	-	-	-
22	Maxmin	0.0000	0.0026	0.0045	0.0134	0.0354	0.0361	0.0505	0.0498	0.0528
23	Minmax	0.0000	0.0026	0.0045	0.0134	0.0361	0.0355	0.0505	0.0495	0.0530
24	Strong q-Par. sim. maj.	0.0000	0.0000	0.0503	0.0282	-	-	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0464	0.0579	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0017	0.0045	0.0140	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_3 (share of profiles in which exactly three voters manipulate)

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8
1	Plurality Rule	0.0000	0.0000	0.0617/0.0620	0.0726/0.0733	0.0999	0.0996	0.0943
2a	Approval Voting, q=2	0.0000	0.0125	0.0276	0.0506	0.0731	0.0877	0.0993
2b	Approval Voting, q=3	0.0000	0.0396	0.0706	0.1069	0.1255	0.1415	0.1519
3	Run-Off Procedure	0.0000	0.0000	0.0351	0.0486	0.0990	0.1385	0.0849
4	Hare’s Procedure	0.0000	0.0000	0.0162	0.0281	0.0630	0.0994	0.1193
5	Inverse Plurality Rule	0.0000	0.0410	0.0980/0.0977	0.1455/0.1460	0.1678	0.1737	0.1728
6	Borda’s Rule	0.0000	0.0028	0.0648/0.0649	0.1965/0.1986	0.2463	0.1815	0.1411
7	Black’s Procedure	0.0000	0.0012	0.0125	0.1056	0.0592	0.0805	0.0756
8	Inverse Borda Proc.	0.0000	0.0028	0.0157	0.0427	0.0851	0.1434	0.1051
9	Nanson’s Procedure	0.0000	0.0001	0.0129	0.0501	0.0716	0.1306	0.0958
10	Coombs’ Procedure	0.0000	0.0157	0.0632	0.1000/0.1005	0.0961	0.0980	0.0879
11	Dominant Set	0.0000	0.0064	0.0714	0.0314	0.0596	0.0494	0.0740
12	Undominated Set	0.0000	0.0064	0.0123	0.0315	0.0586	0.0484	0.0738
13	Min. Weakly Stable Set	0.0000	0.0059	0.0120	0.0322	0.0577	0.0518	0.0750
14	Fishburn’s Procedure	0.0000	0.0013	0.0128	0.0324	0.0596	0.0587	0.0730
15	Uncovered Set I	0.0000	0.0013	0.1330	0.0334	0.1735	0.0603	0.1109
16	Uncovered Set II	0.0000	0.0013	0.0225	0.0330	0.1151	0.0589	0.0914
17	Richelson’s Procedure	0.0000	0.0013	0.1335	0.0330	0.1837	0.0592	0.1100
18	Copeland’s Rule I	0.0000	0.0001	0.1038	0.0480	0.1741	0.0641	0.1191
19	Copeland’s Rule II	0.0000	0.0001	0.0123	0.0469	0.0599	0.0645	0.0736
20	Copeland’s Rule III	0.0000	0.0001	0.0412	0.0473	0.1661	0.0637	0.1126
21	Young’s Procedure	0.0000	0.0047	0.0123	0.0332	-	-	-
22	Maxmin	0.0000	0.0069	0.0124	0.0334	0.0599	0.0619	0.0747
23	Minmax	0.0000	0.0069	0.0123	0.0333	0.0593	0.0604	0.0742
24	Strong q-Par. sim. maj.	0.0000	0.0000	0.064	0.0509	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0494	0.0640	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0049	0.0119	0.0344	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_4 (share of profiles in which exactly four voters manipulate)

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.0000	0.0000	0.0000	0.0000	0.0078	0.0062	0.0129	0.0194	0.0211
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0160	0.0191	0.0158
4	Hare's Procedure	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0171	0.0192	0.0156
5	Inverse Plurality Rule	0.0000	0.0000	0.0023	0.0058	0.0124	0.0212	0.0307	0.0376	0.0458
6	Borda's Rule	0.0000	0.0000	0.0000	0.0000	0.0050	0.0387	0.0423	0.0485	0.0732
7	Black's Procedure	0.0000	0.0000	0.0000	0.0000	0.0000	0.0074	0.0059	0.0118	0.0139
8	Inverse Borda Proc.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0090	0.0449	0.0349
9	Nanson's Procedure	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0059	0.0447	0.0231
10	Coombs' Procedure	0.0000	0.0000	0.0000	0.0000	0.0078	0.0061	0.0080	0.0186	0.0102
11	Dominant Set	0.0000	0.0000	0.0046	0.0000	0.0257	0.0000	0.0241	0.0020	0.0166
12	Undominated Set	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0060	0.0021	0.0094
13	Min. Weakly Stable Set	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0057	0.0020	0.0090
14	Fishburn's Procedure	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0060	0.0019	0.0119
15	Uncovered Set I	0.0000	0.0000	0.0000	0.0000	0.0231	0.0000	0.0382	0.0021	0.0251
16	Uncovered Set II	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0163	0.0020	0.0283
17	Richelson's Procedure	0.0000	0.0000	0.0000	0.0000	0.0231	0.0000	0.0386	0.0021	0.0246
18	Copeland's Rule I	0.0000	0.0000	0.0000	0.0000	0.0233	0.0000	0.0382	0.0019	0.0248
19	Copeland's Rule II	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0059	0.0020	0.0120
20	Copeland's Rule III	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0148	0.0020	0.0257
21	Young's Procedure	0.0000	0.0000	0.0000	0.0000	-	-	-	-	-
22	Maxmin	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0059	0.0063	0.0135
23	Minmax	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0058	0.0065	0.0133
24	Strong q-Par. sim. maj.	0.0000	0.0000	0.0000	0.0077	-	-	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0000	0.0000	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0000	0.0000	0.0000	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_4 (share of profiles in which exactly four voters manipulate)

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.0000	0.0000	0.0000	0.0000	0.0211	0.0370	0.0376	0.0460	0.0468
2a	Approval Voting, q=2	0.0000	0.0000	0.0014	0.0043	0.0089	0.0147	0.0242	0.0314	0.0411
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0000	0.0000	0.0000	0.0010	0.0106	0.0110	0.0416	0.0449	0.0589
4	Hare's Procedure	0.0000	0.0000	0.0000	0.0000	0.0038	0.0088	0.0374	0.0460	0.0418
5	Inverse Plurality Rule	0.0000	0.0000	0.0137	0.0299	0.0455	0.0605	0.0751	0.0842	0.0902
6	Borda's Rule	0.0000	0.0000	0.0008	0.0031	0.0310	0.0967	0.1198	0.1059	0.1006
7	Black's Procedure	0.0000	0.0000	0.0002	0.0027	0.0038	0.0405	0.0185	0.0425	0.0322
8	Inverse Borda Proc.	0.0000	0.0000	0.0005	0.0028	0.0055	0.0127	0.0334	0.0839	0.0648
9	Nanson's Procedure	0.0000	0.0000	0.0002	0.0005	0.0032	0.0111	0.0240	0.0652	0.0518
10	Coombs' Procedure	0.0000	0.0000	0.0020	0.0070	0.0269	0.0345	0.0349	0.0403	0.0282
11	Dominant Set	0.0000	0.0000	0.0116	0.0066	0.0285	0.0114	0.0311	0.0150	0.0301
12	Undominated Set	0.0000	0.0000	0.0002	0.0066	0.0037	0.0116	0.0196	0.0149	0.0296
13	Min. Weakly Stable Set	0.0000	0.0000	0.0002	0.0032	0.0031	0.0077	0.0178	0.0122	0.0290
14	Fishburn's Procedure	0.0000	0.0000	0.0002	0.0013	0.0035	0.0078	0.0187	0.0163	0.0294
15	Uncovered Set I	0.0000	0.0000	0.0013	0.0013	0.0561	0.0078	0.0842	0.0164	0.0566
16	Uncovered Set II	0.0000	0.0000	0.0006	0.0013	0.0067	0.0080	0.0476	0.0163	0.0554
17	Richelson's Procedure	0.0000	0.0000	0.0009	0.0013	0.0579	0.0077	0.0894	0.0166	0.0586
18	Copeland's Rule I	0.0000	0.0000	0.0005	0.0022	0.0515	0.0097	0.0804	0.0192	0.0628
19	Copeland's Rule II	0.0000	0.0000	0.0002	0.0022	0.0036	0.0095	0.0189	0.0188	0.0296
20	Copeland's Rule III	0.0000	0.0000	0.0002	0.0022	0.0109	0.0101	0.0561	0.0189	0.0585
21	Young's Procedure	0.0000	0.0000	0.0002	0.0027	-	-	-	-	-
22	Maxmin	0.0000	0.0000	0.0002	0.0028	0.0036	0.0092	0.0194	0.0201	0.0313
23	Minmax	0.0000	0.0000	0.0002	0.0028	0.0035	0.0096	0.0191	0.0206	0.0312
24	Strong q-Par. sim. maj.	0.0000	0.0000	0.0000	0.0201	-	-	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0030	0.0052	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0000	0.0002	0.0041	-	-	-	-	-

Table 2. Index J of Manipulation for Social Choice Rules

Index J_4 (share of profiles in which exactly four voters manipulate)

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8
1	Plurality Rule	0.0000	0.0000	0.0000	0.0122/0.0122	0.0395	0.0663	0.0681
2a	Approval Voting, q=2	0.0000	0.0000	0.0016	0.0055	0.0151	0.0284	0.0386
2b	Approval Voting, q=3	0.0000	0.0000	0.0078	0.0203	0.0385	0.0556	0.0730
3	Run-Off Procedure	0.0000	0.0000	0.0000	0.0068	0.0295	0.0333	0.0735
4	Hare’s Procedure	0.0000	0.0000	0	0.0015	0.0135	0.0245	0.0634
5	Inverse Plurality Rule	0.0000	0.0000	0.0165/0.0166	0.0497/0.0500	0.0806	0.1026	0.1183
6	Borda’s Rule	0.0000	0.0000	0.0030/0.0028	0.0190/0.0193	0.0762	0.1514	0.1668
7	Black’s Procedure	0.0000	0.0000	0.0007	0.0143	0.0122	0.0809	0.0355
8	Inverse Borda Proc.	0.0000	0.0000	0.0016	0.0098	0.0163	0.0323	0.0615
9	Nanson’s Procedure	0.0000	0.0000	0.0010	0.0093	0.0122	0.0371	0.0488
10	Coombs’ Procedure	0.0000	0.0000	0.0072	0.0271/0.0264	0.0554	0.0712	0.0675
11	Dominant Set	0.0000	0.0000	0.0201	0.0103	0.0431	0.0206	0.0407
12	Undominated Set	0.0000	0.0000	0.0007	0.0102	0.0086	0.0209	0.0331
13	Min. Weakly Stable Set	0.0000	0.0000	0.0007	0.0103	0.0095	0.0209	0.0330
14	Fishburn’s Procedure	0.0000	0.0000	0.0008	0.0052	0.0102	0.0221	0.0334
15	Uncovered Set I	0.0000	0.0000	0.0067	0.0053	0.0982	0.0219	0.1155
16	Uncovered Set II	0.0000	0.0000	0.0022	0.0053	0.0197	0.0212	0.0780
17	Richelson’s Procedure	0.0000	0.0000	0.0058	0.0055	0.1029	0.0219	0.1254
18	Copeland’s Rule I	0.0000	0.0000	0.0035	0.0071	0.0852	0.0311	0.1137
19	Copeland’s Rule II	0.0000	0.0000	0.0008	0.0070	0.0107	0.0311	0.0344
20	Copeland’s Rule III	0.0000	0.0000	0.0013	0.0069	0.0325	0.0314	0.0959
21	Young’s Procedure	0.0000	0.0000	0.0007	0.0074	-	-	-
22	Maxmin	0.0000	0.0000	0.0009	0.0079	0.0103	0.0230	0.0342
23	Minmax	0.0000	0.0000	0.0007	0.0076	0.0106	0.0228	0.0348
24	Strong q-Par. sim. maj.	0.0000	0.0000	0.0007	0.0193	-	-	-
25	Strong q-Par. plurality	0.0000	0.0000	0.0013	0.0142	-	-	-
26	Strongest q-Par. sim. maj	0.0000	0.0000	0.0007	0.0104	-	-	-

Table 3. Index I_1 of Manipulation for Social Choice Rules

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.0222	0.0296	0.0222	0.0247	0.0247	0.0219	0.0222	0.0219	0.0205
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0222	0.0148	0.0222	0.0148	0.0205	0.0145	0.0183	0.0161	0.0167
4	Hare's Procedure	0.0222	0.0148	0.0148	0.0148	0.0136	0.0144	0.0176	0.0163	0.0135
5	Inverse Plurality Rule	0.0667	0.0519	0.0444	0.0420	0.0377	0.0352	0.0334	0.0311	0.0297
6	Borda's Rule	0.0389	0.0185	0.0245	0.0191	0.0196	0.0177	0.0171	0.0162	0.0155
7	Black's Procedure	0.0167	0.0111	0.0134	0.0130	0.0117	0.0126	0.0109	0.0117	0.0102
8	Inverse Borda Proc.	0.0167	0.0111	0.0134	0.0123	0.0121	0.0118	0.0112	0.0114	0.0107
9	Nanson's Procedure	0.0167	0.0111	0.0134	0.0123	0.0117	0.0119	0.0107	0.0112	0.0098
10	Coombs' Procedure	0.0056	0.0185	0.0162	0.0144	0.0125	0.0117	0.0118	0.0144	0.0134
11	Dominant Set	0.0444	0.0111	0.0444	0.0120	0.0415	0.0117	0.0385	0.0111	0.0361
12	Undominated Set	0.0167	0.0111	0.0134	0.0120	0.0122	0.0116	0.0114	0.0112	0.0107
13	Min. Weakly Stable Set	0.0167	0.0111	0.0134	0.0120	0.0121	0.0117	0.0113	0.0112	0.0107
14	Fishburn's Procedure	0.0167	0.0111	0.0134	0.0120	0.0116	0.0117	0.0106	0.0110	0.0098
15	Uncovered Set I	0.0389	0.0111	0.0338	0.0120	0.0298	0.0115	0.0267	0.0111	0.0250
16	Uncovered Set II	0.0167	0.0111	0.0181	0.0120	0.0176	0.0117	0.0164	0.0112	0.0156
17	Richelson's Procedure	0.0389	0.0111	0.0338	0.0120	0.0297	0.0117	0.0271	0.0111	0.0249
18	Copeland's Rule I	0.0389	0.0111	0.0338	0.0120	0.0299	0.0117	0.0269	0.0112	0.0249
19	Copeland's Rule II	0.0167	0.0111	0.0134	0.0120	0.0117	0.0118	0.0106	0.0112	0.0099
20	Copeland's Rule III	0.0389	0.0111	0.0245	0.0120	0.0186	0.0118	0.0153	0.0111	0.0135
21	Young's Procedure	0.0167	0.0111	0.0139	0.0108	-	-	-	-	-
22	Maxmin	0.0167	0.0111	0.0134	0.0108	0.0118	0.0100	0.0106	0.0094	0.0096
23	Minmax	0.0167	0.0111	0.0134	0.0108	0.0117	0.0100	0.0105	0.0093	0.0096
24	Strong q-Par. sim. maj.	0.0333	0.0222	0.0287	0.0250	-	-	-	-	-
25	Strong q-Par. plurality	0.0333	0.0111	0.0315	0.0194	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0167	0.0111	0.0134	0.0120	-	-	-	-	-

Table 3. Index I_1 of Manipulation for Social Choice Rules

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.0326	0.0408	0.0367	0.0336	0.0344	0.0334	0.0316	0.0303	0.0299
2a	Approval Voting, q=2	0.0580	0.0451	0.0419	0.0376	0.0351	0.0327	0.0311	0.0295	0.0283
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0326	0.0251	0.0326	0.0260	0.0290	0.0279	0.0269	0.0254	0.0258
4	Hare's Procedure	0.0326	0.0251	0.0251	0.0251	0.0249	0.0256	0.0292	0.0274	0.0236
5	Inverse Plurality Rule	0.0652	0.0652	0.0591	0.0530	0.0487	0.0461	0.0438	0.0414	0.0393
6	Borda's Rule	0.0424	0.0290	0.0301	0.0272	0.0258	0.0243	0.0233	0.0222	0.0214
7	Black's Procedure	0.0257	0.0182	0.0211	0.0199	0.0190	0.0192	0.0178	0.0184	0.0168
8	Inverse Borda Proc.	0.0257	0.0181	0.0217	0.0204	0.0200	0.0200	0.0189	0.0191	0.0181
9	Nanson's Procedure	0.0257	0.0181	0.0212	0.0190	0.0191	0.0181	0.0178	0.0172	0.0168
10	Coombs' Procedure	0.0093	0.0262	0.0254	0.0234	0.0217	0.0220	0.0214	0.0233	0.0226
11	Dominant Set	0.0480	0.0268	0.0449	0.0287	0.0408	0.0276	0.0373	0.0262	0.0346
12	Undominated Set	0.0257	0.0268	0.0211	0.0287	0.0194	0.0277	0.0186	0.0262	0.0181
13	Min. Weakly Stable Set	0.0257	0.0199	0.0211	0.0223	0.0189	0.0219	0.0176	0.0212	0.0167
14	Fishburn's Procedure	0.0257	0.0180	0.0211	0.0204	0.0190	0.0203	0.0176	0.0196	0.0167
15	Uncovered Set I	0.0424	0.0180	0.0452	0.0204	0.0429	0.0203	0.0405	0.0195	0.0381
16	Uncovered Set II	0.0257	0.0180	0.0296	0.0204	0.0291	0.0203	0.0281	0.0196	0.0265
17	Richelson's Procedure	0.0424	0.0180	0.0447	0.0204	0.0427	0.0201	0.0402	0.0196	0.0381
18	Copeland's Rule I	0.0424	0.0173	0.0422	0.0195	0.0394	0.0195	0.0369	0.0189	0.0346
19	Copeland's Rule II	0.0257	0.0173	0.0211	0.0195	0.0190	0.0194	0.0176	0.0189	0.0166
20	Copeland's Rule III	0.0424	0.0173	0.0305	0.0195	0.0249	0.0194	0.0218	0.1900	0.0199
21	Young's Procedure	0.0257	0.0206	0.0211	0.0199	-	-	-	-	-
22	Maxmin	0.0257	0.0208	0.0211	0.0199	0.0191	0.0183	0.0176	0.0170	0.0164
23	Minmax	0.0257	0.0208	0.0211	0.0199	0.0192	0.0184	0.0176	0.0169	0.0164
24	Strong q-Par. sim. maj.	0.0326	0.0302	0.0373	0.0336	-	-	-	-	-
25	Strong q-Par. plurality	0.0326	0.0270	0.0337	0.0321	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0257	0.0186	0.0211	0.0211	-	-	-	-	-

Table 3. Index I_1 of Manipulation for Social Choice Rules

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	5.10	5.51
1	Plurality Rule	0.0403	0.0484	0.0465/0.0465	0.0422/0.0422	0.0404	0.0404	0.0391	0.0376	0.0359	0.0195
2a	Approval Voting, q=2	0.0575	0.0454	0.0443	0.0399	0.0383	0.0361	0.0343	0.0328	0.0315	0.0161
2b	Approval Voting, q=3	0.0706	0.0627	0.0541	0.0499	0.0465	0.0431	0.0410	0.0387	0.0370	0.0172
3	Run-Off Procedure	0.0403	0.0336	0.0402	0.0352	0.0360	0.0369	0.0349	0.0345	0.0327	0.0210
4	Hare's Procedure	0.0403	0.0336	0.0335	0.0337	0.0347	0.0356	0.0398	0.0376	0.0338	0.0239
5	Inverse Plurality Rule	0.0605	0.0605	0.0605/0.0605	0.0582/0.0583	0.0548	0.0513	0.0484	0.0461	0.0441	0.0211
6	Borda's Rule	0.0426	0.0356	0.0345/0.0345	0.0323/0.0325	0.0307	0.0290	0.0278	0.0265	0.0256	0.0128
7	Black's Procedure	0.0327	0.0232	0.0273	0.0253	0.0251	0.0245	0.0236	0.0237	0.0225	0.0143
8	Inverse Borda Proc.	0.0327	0.0240	0.0286	0.0273	0.0268	0.0269	0.0257	0.0260	0.0247	0.0157
9	Nanson's Procedure	0.0327	0.0241	0.0278	0.0262	0.0255	0.0256	0.0240	0.0243	0.0228	0.0139
10	Coombs' Procedure	0.0125	0.0318	0.0328	0.0321	0.0303	0.0309	0.0298	0.0323	0.0313	0.0219
11	Dominant Set	0.0510	0.0289	0.0496	0.0330	0.0466	0.0327	0.0434	0.0319	0.0407	0.0188
12	Undominated Set	0.0327	0.0289	0.0275	0.0329	0.0250	0.0327	0.0237	0.0315	0.0228	0.0189
13	Min. Weakly Stable Set	0.0327	0.0285	0.0274	0.0327	0.0250	0.0322	0.0238	0.0314	0.0228	0.0190
14	Fishburn's Procedure	0.0327	0.0240	0.0274	0.0279	0.0251	0.0281	0.0234	0.0276	0.0225	0.0170
15	Uncovered Set I	0.0429	0.0240	0.0541	0.0280	0.0538	0.0283	0.0517	0.0276	0.0497	0.0170
16	Uncovered Set II	0.0327	0.0240	0.0225	0.0281	0.0391	0.0281	0.0380	0.0276	0.0364	0.0171
17	Richelson's Procedure	0.0429	0.0240	0.0530	0.0281	0.0534	0.0282	0.0512	0.0274	0.0491	0.0172
18	Copeland's Rule I	0.0426	0.0219	0.0473	0.0256	0.0452	0.0260	0.0431	0.0253	0.0413	0.0168
19	Copeland's Rule II	0.0327	0.0219	0.0273	0.0254	0.0251	0.0259	0.0236	0.0257	0.0225	0.0170
20	Copeland's Rule III	0.0426	0.0219	0.0344	0.0255	0.0296	0.0259	0.0267	0.0257	0.0247	0.0169
21	Young's Procedure	0.0327	0.0292	0.0273	0.028	-	-	-	-	-	-
22	Maxmin	0.0327	0.0297	0.0273	0.0281	0.0254	0.0259	0.0237	0.0239	0.0225	0.0124
23	Minmax	0.0327	0.0297	0.0272	0.028	0.0253	0.0256	0.0238	0.0238	0.0225	0.0123
24	Strong q-Par. sim. maj.	0.0340	0.0367	0.0427	0.0406	-	-	-	-	-	-
25	Strong q-Par. plurality	0.0340	0.0343	0.0401	0.0368	-	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0327	0.0243	0.0272	0.0284	-	-	-	-	-	-

Table 4. Index I_2 of Manipulation for Social Choice Rules

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.0556	0.0741	0.0556	0.0617	0.0617	0.0548	0.0556	0.0547	0.0514
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0556	0.0370	0.0556	0.0370	0.0513	0.0362	0.0458	0.0405	0.0416
4	Hare's Procedure	0.0556	0.0370	0.0370	0.0370	0.0339	0.0360	0.0439	0.0407	0.0337
5	Inverse Plurality Rule	0.1667	0.1296	0.1111	0.1049	0.0944	0.0879	0.0835	0.0779	0.0744
6	Borda's Rule	0.1944	0.0926	0.1227	0.0957	0.0979	0.0883	0.0856	0.0808	0.0777
7	Black's Procedure	0.0556	0.0370	0.0440	0.0448	0.0371	0.0425	0.0337	0.0390	0.0308
8	Inverse Borda Proc.	0.0556	0.0370	0.0440	0.0478	0.0417	0.0456	0.0402	0.0434	0.0386
9	Nanson's Procedure	0.0556	0.0370	0.0440	0.0478	0.0371	0.0460	0.0337	0.0423	0.0311
10	Coombs' Procedure	0.0278	0.1111	0.0787	0.0725	0.0579	0.0538	0.0515	0.0613	0.0556
11	Dominant Set	0.1111	0.0370	0.0926	0.0340	0.0814	0.0304	0.0735	0.0274	0.0674
12	Undominated Set	0.0556	0.0370	0.0440	0.0340	0.0369	0.0303	0.0326	0.0275	0.0292
13	Min. Weakly Stable Set	0.0556	0.0370	0.0440	0.0340	0.0366	0.0304	0.0323	0.0276	0.0291
14	Fishburn's Procedure	0.0556	0.0370	0.0440	0.0340	0.0367	0.0302	0.0324	0.0273	0.0290
15	Uncovered Set I	0.1944	0.0370	0.1366	0.0340	0.1081	0.0299	0.0903	0.0275	0.0799
16	Uncovered Set II	0.0556	0.0370	0.0579	0.0340	0.0543	0.0303	0.0493	0.0277	0.0461
17	Richelson's Procedure	0.1944	0.0370	0.1366	0.0340	0.1076	0.0304	0.0914	0.0274	0.0797
18	Copeland's Rule I	0.1944	0.0370	0.1366	0.0340	0.1083	0.0303	0.0910	0.0277	0.0795
19	Copeland's Rule II	0.0556	0.0370	0.0440	0.0340	0.0372	0.0305	0.0326	0.0275	0.0292
20	Copeland's Rule III	0.1944	0.0370	0.1019	0.0340	0.0702	0.0305	0.0545	0.0273	0.0455
21	Young's Procedure	0.0556	0.0370	0.0440	0.0347	-	-	-	-	-
22	Maxmin	0.0556	0.0370	0.0440	0.0347	0.0373	0.0311	0.0328	0.0287	0.0291
23	Minmax	0.0556	0.0370	0.0440	0.0347	0.0371	0.0312	0.0324	0.0286	0.0292
24	Strong q-Par. sim. maj.	0.0833	0.0648	0.0764	0.0687	-	-	-	-	-
25	Strong q-Par. plurality	0.0833	0.0370	0.0833	0.0556	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0556	0.0370	0.0440	0.0340	-	-	-	-	-

Table 4. Index I_2 of Manipulation for Social Choice Rules

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.1354	0.1771	0.1523	0.1484	0.1539	0.1488	0.1408	0.1381	0.1368
2a	Approval Voting, q=2	0.2083	0.1620	0.1470	0.1292	0.1196	0.1100	0.1041	0.0976	0.0935
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.1354	0.0990	0.1406	0.1083	0.1253	0.1132	0.1183	0.1063	0.1135
4	Hare's Procedure	0.1354	0.0990	0.1009	0.0987	0.0943	0.0968	0.1107	0.1039	0.0907
5	Inverse Plurality Rule	0.3333	0.3333	0.3021	0.2708	0.2488	0.2362	0.2239	0.2117	0.2010
6	Borda's Rule	0.3424	0.2499	0.2526	0.2299	0.2184	0.2057	0.1969	0.1876	0.1799
7	Black's Procedure	0.1308	0.1255	0.1046	0.1297	0.0908	0.1199	0.0836	0.1110	0.0779
8	Inverse Borda Proc.	0.1308	0.0984	0.1101	0.1196	0.1074	0.1158	0.1043	0.1090	0.1005
9	Nanson's Procedure	0.1308	0.0973	0.1046	0.1079	0.0949	0.1005	0.0897	0.0937	0.0850
10	Coombs' Procedure	0.0677	0.2428	0.2097	0.1872	0.1662	0.1629	0.1516	0.1610	0.1513
11	Dominant Set	0.2448	0.1044	0.1744	0.1021	0.1462	0.0931	0.1283	0.0855	0.1157
12	Undominated Set	0.1308	0.1044	0.1046	0.1021	0.0911	0.0933	0.0824	0.0854	0.0762
13	Min. Weakly Stable Set	0.1308	0.0988	0.1046	0.0940	0.0898	0.0848	0.0808	0.0779	0.0733
14	Fishburn's Procedure	0.1308	0.0974	0.1046	0.0913	0.0908	0.0828	0.0812	0.0757	0.0743
15	Uncovered Set I	0.3424	0.0974	0.2784	0.0913	0.2299	0.0828	0.1992	0.0753	0.1768
16	Uncovered Set II	0.1308	0.0974	0.1414	0.0913	0.1333	0.0828	0.1250	0.0758	0.1155
17	Richelson's Procedure	0.3424	0.0974	0.2814	0.0913	0.2355	0.0823	0.2041	0.0757	0.1823
18	Copeland's Rule I	0.3424	0.1003	0.2679	0.0939	0.2218	0.0850	0.1918	0.0771	0.1704
19	Copeland's Rule II	0.1308	0.1003	0.1046	0.0939	0.0912	0.0844	0.0816	0.0771	0.0745
20	Copeland's Rule III	0.3424	0.1003	0.2095	0.0939	0.1573	0.0847	0.1295	0.0773	0.1116
21	Young's Procedure	0.1308	0.1034	0.1046	0.0972	-	-	-	-	-
22	Maxmin	0.1308	0.1038	0.1046	0.0973	0.0911	0.0873	0.0819	0.0791	0.0751
23	Minmax	0.1308	0.1038	0.1046	0.0973	0.0913	0.0874	0.0818	0.0789	0.0752
24	Strong q-Par. sim. maj.	0.1944	0.1315	0.1432	0.1313	-	-	-	-	-
25	Strong q-Par. plurality	0.1944	0.1109	0.1476	0.1257	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.1308	0.0965	0.1046	0.0929	-	-	-	-	-

Table 4. Index I_2 of Manipulation for Social Choice Rules

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8
1	Plurality Rule	0.2283	0.2960	0.2724/0.2731	0.2586/0.2588	0.2605	0.2625	0.2535
2a	Approval Voting, q=2	0.2600	0.2117	0.1924	0.1725	0.1635	0.1529	0.1453
2b	Approval Voting, q=3	0.4833	0.3703	0.3118	0.2847	0.2559	0.2356	0.2204
3	Run-Off Procedure	0.2283	0.1773	0.2439	0.2024	0.2208	0.2086	0.2165
4	Hare's Procedure	0.2283	0.1773	0.1845	0.1771	0.1741	0.1767	0.1968
5	Inverse Plurality Rule	0.5000	0.5000	0.5000/0.5001	0.4808/0.4821	0.4528	0.4240	0.4002
6	Borda's Rule	0.4539	0.4010	0.3893/0.3883	0.3666/0.3683	0.3483	0.3309	0.3167
7	Black's Procedure	0.2148	0.2259	0.1745	0.2315	0.1561	0.2146	0.1449
8	Inverse Borda Proc.	0.2148	0.1770	0.1892	0.2068	0.1881	0.1999	0.1845
9	Nanson's Procedure	0.2148	0.1857	0.1787	0.2088	0.1676	0.1986	0.1610
10	Coombs' Procedure	0.1138	0.3734	0.3683	0.3403/0.3396	0.3121	0.3119	0.2890
11	Dominant Set	0.3905	0.1772	0.2732	0.1741	0.2314	0.1598	0.2045
12	Undominated Set	0.2148	0.1772	0.1751	0.1744	0.1527	0.1596	0.1384
13	Min. Weakly Stable Set	0.2148	0.1785	0.1741	0.1756	0.1529	0.1604	0.1392
14	Fishburn's Procedure	0.2148	0.1731	0.1746	0.1656	0.1546	0.1524	0.1388
15	Uncovered Set I	0.4594	0.1731	0.4219	0.1663	0.3639	0.1531	0.3195
16	Uncovered Set II	0.2148	0.1731	0.2406	0.1662	0.2270	0.1516	0.2137
17	Richelson's Procedure	0.4594	0.1731	0.4281	0.1664	0.3754	0.1525	0.3322
18	Copeland's Rule I	0.4539	0.1767	0.3895	0.1711	0.3351	0.1560	0.2959
19	Copeland's Rule II	0.2148	0.1767	0.1740	0.1701	0.1544	0.1558	0.1408
20	Copeland's Rule III	0.4539	0.1767	0.3189	0.1704	0.2558	0.1555	0.2159
21	Young's Procedure	0.2148	0.1846	0.1739	0.1745	-	-	-
22	Maxmin	0.2148	0.1860	0.1739	0.174	0.1548	0.1561	0.1409
23	Minmax	0.2148	0.1860	0.1731	0.1736	0.1549	0.1551	0.1410
24	Strong q-Par. sim. maj.	0.2250	0.1975	0.2044	0.1917	-	-	-
25	Strong q-Par. plurality	0.2250	0.1828	0.2066	0.1896	-	-	-
26	Strongest q-Par. sim. maj	0.2148	0.1650	0.1730	0.1647	-	-	-

Table 5. Index I_3 of Manipulation for Social Choice Rules

	(m.n)	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	3.10
1	Plurality Rule	0.0556	0.0741	0.0556	0.0617	0.0625	0.0548	0.0560	0.0547	0.0513
2a	Approval Voting, q=2	-	-	-	-	-	-	-	-	-
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.0556	0.0370	0.0556	0.0370	0.0514	0.0361	0.0456	0.0404	0.0412
4	Hare's Procedure	0.0556	0.0370	0.0370	0.0370	0.0344	0.0362	0.0438	0.0404	0.0337
5	Inverse Plurality Rule	0.1667	0.1296	0.1111	0.1049	0.0945	0.0873	0.0832	0.0776	0.0737
6	Borda's Rule	0.1944	0.0926	0.1227	0.0957	0.0975	0.0881	0.0860	0.0811	0.0779
7	Black's Procedure	0.0556	0.0370	0.0440	0.0448	0.0372	0.0425	0.0329	0.0392	0.0310
8	Inverse Borda Proc.	0.0556	0.0370	0.0440	0.0478	0.0418	0.0479	0.0407	0.0444	0.0391
9	Nanson's Procedure	0.0556	0.0370	0.0440	0.0478	0.0370	0.0479	0.0339	0.0440	0.0316
10	Coombs' Procedure	0.0278	0.1111	0.0787	0.0725	0.0579	0.0534	0.0515	0.0606	0.0557
11	Dominant Set	0.1111	0.0370	0.0926	0.0340	0.0816	0.0303	0.0735	0.0273	0.0679
12	Undominated Set	0.0556	0.0370	0.0440	0.0340	0.0369	0.0301	0.0327	0.0277	0.0293
13	Min. Weakly Stable Set	0.0556	0.0370	0.0440	0.0340	0.0373	0.0304	0.0324	0.0274	0.0292
14	Fishburn's Procedure	0.0556	0.0370	0.0440	0.0340	0.0367	0.0304	0.0324	0.0274	0.0295
15	Uncovered Set I	0.1944	0.0370	0.1366	0.0340	0.1083	0.0303	0.0914	0.0275	0.0798
16	Uncovered Set II	0.0556	0.0370	0.0579	0.0340	0.0539	0.0303	0.0496	0.0273	0.0459
17	Richelson's Procedure	0.1944	0.0370	0.1366	0.0340	0.1086	0.0305	0.0908	0.0275	0.0799
18	Copeland's Rule I	0.1944	0.0370	0.1366	0.0340	0.1083	0.0303	0.0908	0.0274	0.0788
19	Copeland's Rule II	0.0556	0.0370	0.0440	0.0340	0.0369	0.0304	0.0327	0.0277	0.0293
20	Copeland's Rule III	0.1944	0.0370	0.1019	0.0340	0.0706	0.0303	0.0547	0.0275	0.0455
21	Young's Procedure	0.0556	0.0370	0.0440	0.0347	-	-	-	-	-
22	Maxmin	0.0556	0.0370	0.0440	0.0347	0.0372	0.0317	0.0324	0.0285	0.0295
23	Minmax	0.0556	0.0370	0.0440	0.0347	0.0367	0.0314	0.0327	0.0285	0.0293
24	Strong q-Par. sim. maj.	0.0833	0.0648	0.0764	0.0687	-	-	-	-	-
25	Strong q-Par. plurality	0.0833	0.0370	0.0833	0.0556	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.0556	0.0370	0.0440	0.0340	-	-	-	-	-

Table 5. Index I_3 of Manipulation for Social Choice Rules

	(m.n)	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	4.10
1	Plurality Rule	0.1465	0.1880	0.1640	0.1566	0.1614	0.1564	0.1478	0.1450	0.1426
2a	Approval Voting, q=2	0.2083	0.1620	0.1470	0.1292	0.1202	0.1103	0.1045	0.0978	0.0932
2b	Approval Voting, q=3	-	-	-	-	-	-	-	-	-
3	Run-Off Procedure	0.1459	0.1042	0.1487	0.1117	0.1304	0.1178	0.1240	0.1099	0.1170
4	Hare's Procedure	0.1456	0.1042	0.1043	0.1030	0.0993	0.1009	0.1157	0.1079	0.0940
5	Inverse Plurality Rule	0.3318	0.3332	0.3012	0.2702	0.2480	0.2353	0.2221	0.2119	0.2007
6	Borda's Rule	0.3757	0.2657	0.2668	0.2410	0.2289	0.2153	0.2053	0.1941	0.1858
7	Black's Procedure	0.1386	0.1383	0.1088	0.1393	0.0953	0.1269	0.0868	0.1172	0.0809
8	Inverse Borda Proc.	0.1396	0.1070	0.1158	0.1320	0.1133	0.1256	0.1111	0.1177	0.1057
9	Nanson's Procedure	0.1387	0.1094	0.1086	0.1192	0.0999	0.1089	0.0939	0.1006	0.0892
10	Coombs' Procedure	0.0716	0.2534	0.2214	0.1970	0.1750	0.1723	0.1582	0.1687	0.1578
11	Dominant Set	0.2712	0.1119	0.1867	0.1075	0.1535	0.0963	0.1350	0.0886	0.1198
12	Undominated Set	0.1387	0.1120	0.1092	0.1072	0.0952	0.0970	0.0854	0.0882	0.0782
13	Min. Weakly Stable Set	0.1379	0.1051	0.1093	0.0981	0.0941	0.0879	0.0831	0.0802	0.0753
14	Fishburn's Procedure	0.1395	0.1045	0.1098	0.0959	0.0955	0.0865	0.0838	0.0789	0.0768
15	Uncovered Set I	0.3748	0.1027	0.2967	0.0956	0.2430	0.0872	0.2082	0.0795	0.1840
16	Uncovered Set II	0.1384	0.1040	0.1507	0.0962	0.1415	0.0871	0.1306	0.0793	0.1209
17	Richelson's Procedure	0.3746	0.1042	0.2997	0.0958	0.2470	0.0868	0.2142	0.0786	0.1900
18	Copeland's Rule I	0.3747	0.1052	0.2847	0.0975	0.2319	0.0873	0.1993	0.0804	0.1771
19	Copeland's Rule II	0.1391	0.1058	0.1094	0.0980	0.0946	0.0878	0.0843	0.0792	0.0768
20	Copeland's Rule III	0.3739	0.1061	0.2205	0.0989	0.1639	0.0877	0.1341	0.0796	0.1146
21	Young's Procedure	0.1385	0.1094	0.1091	0.1017	-	-	-	-	-
22	Maxmin	0.1392	0.1111	0.1096	0.1013	0.0943	0.0899	0.0852	0.0816	0.0770
23	Minmax	0.1391	0.1116	0.1087	0.1023	0.0945	0.0897	0.0845	0.0818	0.0771
24	Strong q-Par. sim. maj.	0.1946	0.1368	0.1443	0.1325	-	-	-	-	-
25	Strong q-Par. plurality	0.1946	0.1154	0.1473	0.1270	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.1403	0.1014	0.1094	0.0961	-	-	-	-	-

Table 5. Index I_3 of Manipulation for Social Choice Rules

	(m.n)	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	5.10
1	Plurality Rule	0.2611	0.3279	0.3063	0.2871	0.2854	0.2845	0.2759	0.2644	0.2580
2a	Approval Voting, q=2	0.2650	0.2141	0.1960	0.1777	0.1669	0.1557	0.1482	0.1413	0.1353
2b	Approval Voting, q=3	0.5000	0.3797	0.3207	0.2923	0.2621	0.2409	0.2250	0.2097	0.1982
3	Run-Off Procedure	0.2607	0.1941	0.2675	0.2194	0.2389	0.2221	0.2310	0.2146	0.2185
4	Hare's Procedure	0.2583	0.1946	0.1984	0.1913	0.1855	0.1891	0.2084	0.1991	0.1807
5	Inverse Plurality Rule	0.4977	0.5016	0.4993	0.4820	0.4522	0.4235	0.3985	0.3806	0.3680
6	Borda's Rule	0.5359	0.4514	0.4335	0.4027	0.3790	0.3601	0.3400	0.3263	0.3109
7	Black's Procedure	0.2393	0.2654	0.1899	0.2630	0.1680	0.2370	0.1559	0.2188	0.1469
8	Inverse Borda Proc.	0.2391	0.2053	0.2112	0.2405	0.2092	0.2296	0.2039	0.2137	0.1977
9	Nanson's Procedure	0.2384	0.2272	0.1948	0.2444	0.1830	0.2247	0.1762	0.2060	0.1687
10	Coombs' Procedure	0.1246	0.4079	0.4101	0.3788	0.3421	0.3429	0.3153	0.3281	0.3099
11	Dominant Set	0.4401	0.1979	0.2963	0.1878	0.2468	0.1701	0.2156	0.1550	0.1955
12	Undominated Set	0.2385	0.1974	0.1878	0.1882	0.1621	0.1694	0.1459	0.1558	0.1349
13	Min. Weakly Stable Set	0.2398	0.1989	0.1879	0.1903	0.1632	0.1722	0.1484	0.1578	0.1354
14	Fishburn's Procedure	0.2382	0.1946	0.1907	0.1844	0.1660	0.1658	0.1490	0.1512	0.1369
15	Uncovered Set I	0.5428	0.1947	0.475	0.1838	0.4021	0.1665	0.3502	0.1512	0.3139
16	Uncovered Set II	0.2387	0.1945	0.2678	0.1834	0.2509	0.1670	0.2347	0.1505	0.2188
17	Richelson's Procedure	0.5446	0.1955	0.4846	0.1837	0.4157	0.1660	0.3660	0.1512	0.3277
18	Copeland's Rule I	0.5349	0.1957	0.4351	0.1853	0.3658	0.1657	0.3188	0.1517	0.2857
19	Copeland's Rule II	0.2378	0.1949	0.1889	0.1839	0.1665	0.1658	0.1488	0.1513	0.1372
20	Copeland's Rule III	0.5369	0.1955	0.3519	0.1852	0.2763	0.1659	0.2297	0.1524	0.2005
21	Young's Procedure	0.2392	0.2069	0.1890	0.1891	-	-	-	-	-
22	Maxmin	0.2399	0.2075	0.1885	0.1876	0.1663	0.1661	0.1508	0.1499	0.1388
23	Minmax	0.2398	0.2081	0.1891	0.1881	0.1651	0.1654	0.1503	0.1494	0.1385
24	Strong q-Par. sim. maj.	0.2251	0.2108	0.2066	0.1970	-	-	-	-	-
25	Strong q-Par. plurality	0.2248	0.1957	0.2076	0.1932	-	-	-	-	-
26	Strongest q-Par. sim. maj	0.2389	0.1773	0.1901	0.1752	-	-	-	-	-