The Effects of Volatility on Growth and Financial Development through Capital Market Imperfections

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Abstract
This paper provides a model to account for the empirical evidence that volatility reduces growth. In the model, greater volatility increases the cost associated with capital market imperfections and induces the financial intermediaries to charge higher interest rates. The model is based on one of overlapping generations with two types of technologies. The more productive technology requires fixed investment in the first period. Individual with income less than the amount of fixed investment may borrow in financial markets to obtain more productive technology. Increase in volatility raises the cost of borrowing and makes it less attractive to invest in more productive technology for individuals below certain income in the first period. Hence, volatility reduces growth by deterring people from taking advantage of more productive technology. This model also explains the empirical findings of Ramey and Ramey (1995) that investment is not the channel between volatility and growth by suggesting that totals factor productivity rather than the total factor accumulation is the key for growth.

**Keywords:** Volatility, Growth, Financial Development, Capital Market Imperfections, Costly State Verification, Limited Enforceability of Contracts.

**JEL Classifications:** E220, G200, G330, O110, O400

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1-Introduction

Distinctions between concepts of growth and development appear to be more relevant for many developing countries, considering their unstable growth experiences. Developing countries generally experience long period of economic booms followed by severe crises and recessions. Therefore, high growth performances experienced by many developing countries may not bring high level of development by taking proceeding slow down of their economies into account. High and positive growth rates must be stable for development. (Betancourt 1996). From this perspective, development rather than the growth is the primary objective by being a statement about the sustained growth.

Unstable growth experiences of many developing countries become more relevant if the volatility of growth effects the long-term development. The recent studies provide evidence in this direction by indicating a negative relationship between volatility and growth [like Ramey and Ramey (1995) Aizenman and Marion (1997) Mobarak (2005)]. In this paper, we analyze the volatility and growth relationship by exploring the role of volatility on the losses associated with capital market imperfections. The motivation of the paper is based on an observation that countries with high growth volatility are also characterized with lack of well-developed financial markets and high degree of capital market imperfections. In this paper, we provide a model to explain this observation. The model shows that higher volatility first aggravates cost of capital market imperfections and then the increase in realized financial market imperfections prevents some people
from benefiting the more productive technology. In other words, volatility reduces growth by adversely effecting financial markets.

Attempts to unravel the relationship between volatility and growth require to ask what is the mechanism linking the volatility to growth. The current literature mainly emphasizes the investment as a channel from volatility to growth. In this paper, however, we point out the productivity of investment rather than the level of investment as a primary reason for the adverse effect of volatility on growth. Our model suggests that volatility reduces the total productivity of an economy by aggravating the financial market imperfections and preventing the people from obtaining more productive technologies.

Earlier research on volatility and growth focuses more on the relationship between volatility and investment due to the idea that increase in investment leads to higher growth. Theoretical literature provides explanations for both positive and negative relationship between volatility and investment. The positive link suggests that higher volatility increases the saving rate and thereby promotes the level of investment due to the precautionary motive. Another argument related with precautionary motive is that higher uncertainty associated with higher volatility induces people to acquire more human capital to hedge against future income uncertainty [Canton (2002)].

There seems to be more reasons to believe that volatility and investment could be negatively related. Irreversibilities in investment support the negative relationship between volatility and investment [Bernanke (1989), Pindyck and Solimano (1994), Ranciere et al. (2003) and Aizenman and Marion (1993)]. These models imply that
volatility can reduce capital investment when adjustment costs are asymmetric and hence the investments are irreversible.

Empirical evidence on volatility and growth is also mixed. Kormendi and Meguire (1985) and Grier and Tullock (1989) find positive relationship between volatility of output growth and the mean growth rate using cross country comparison. Empirical evidence on negative relationship between volatility and growth seems to be more substantiated. Ramey and Ramey (1995) demonstrate a negative link between volatility and growth by using a panel of 92 countries as well as a subset of OECD countries. The findings of Aizenman and Marion (1993), (1997) and Mobarak (2005) also confirm the negative relationship between volatility and development.

One surprising finding of Ramey and Ramey (1995) is that volatility lowers growth but is not significantly related to investment. Then, one may ask if investment is not the factor linking volatility to growth, what else accounts for the strong empirical evidence on the negative relationship between volatility and growth? The most likely answer is the factor productivity. Volatility hinders growth not only by reducing the level of investment but by adversely affecting the productivity of production. The significance of total factor productivity in growth is documented by Easterly and Levine (2001). They show that factor accumulation like investment on capital accumulation does not account for the cross-country differences in growth rates and they conclude that total factor productivity accounts for a substantial amount of cross-country growth differences. In this respect, our model provides a better explanation for the effects of volatility on the total factor productivity. In our model there are two types of technologies and volatility
prevents the people from obtaining more productive technology by aggravating the capital market imperfections.

The paper is organized as follows. In the next section, the model is specified and explained in details. Hence, section 2 first exposes the definition of volatility employed in this paper and then it elucidates the basic propositions in the adverse effects of volatility on financial development. In section 3, the effects of volatility on financial development are linked to growth. Lastly, section 4 concludes. All the proofs are delegated to the appendix.

2-The Model

2-1-Production Technologies and Preferences:

A small open economy has two production technologies to produce one good. More advanced technology uses fixed (human)\(^1\) capital and an inferior technology uses unskilled labor. Production with advanced technology is denoted by:

\[
Y_t^S = F(A, I) = AI \\
A = (1 + \delta + \epsilon_i) a \\
a \geq 1
\]

(1)

where \(Y_t^S\) is output at time \(t\) and \(A\) is the productivity factor and \(I\) is fixed investment on human capital needed to use this technology. To introduce the volatility into the model two types of productivity shocks are introduced. \(\epsilon_i\) is idiosyncratic productivity shock affecting individual \(i\) and independently and identically distributed across all individuals.

\(^1\) Throughout the model, the fixed capital and human capital are used interchangeably. The fixed capital investment is required in the model. Whether it is human capital or any other types of capital is not crucial for the implications of the model.
with cumulative density function $F(\varepsilon_i)$ between $-\tilde{\varepsilon} \leq \varepsilon_i \leq \tilde{\varepsilon}$. Aggregate macroeconomic shock is represented by $\delta$ which is assumed to take only two values.

$\delta = \begin{cases} +\delta \text{...with...prob...0.5} \\ -\delta \text{...with...prob...0.5} \end{cases}$

Therefore, aggregate volatility $\delta$ has zero mean. Negative realization of $\delta$ then characterizes a “recession” and positive realizations represent a “boom”. For the sake of simplicity fixed investment on human capital gives constant return to scale without an adjustment cost. Inferior technology is described by:

$$Y_i^N = Bw_n L_i^n \quad B = (1 + \delta + \varepsilon_i)$$

(2)

where $Y_i^N$ and $L_i^n$ are output and unskilled labor input respectively. $B$ represents the volatile productivity parameter and $Bw_n$ is the marginal productivity of unskilled labor in this sector.

The model is based on a two-period overlapping generations. The young decide whether to invest in human capital so that they can obtain the advanced technology and more output in the second period when they are old. Otherwise, individuals work in unskilled technology in both periods. Investment on human capital is fixed, indivisible represented by $I$. Each individual has one unit of labor in each period. Each individual has a parent when they are young and a child when they are old such that the population growth is zero. Individuals are altruistic and leave bequests to their children. To simplify the consumption and saving decision, it is assumed that all the consumption takes place in the second period of life. Hence, utility of an individual is denoted as:

$$U = \alpha \log c + (1 - \alpha) \log b$$

(3)
where $c$ is consumption in the second period and $b$ denotes the bequest and $0 < \alpha < 1$. Since all individuals have one unit of labor, income distribution in the first period is determined by the distribution of bequests.

2-2-Financial Markets:

Individuals have access to capital markets with some capital market imperfections. The world interest rate is equal to $r$ and constant over time. Individuals lend with the world interest rate to the banks while the cost of borrowing is higher than $r$ due capital market imperfections. Capital market imperfections are modeled under costly state verification framework [Towsend (1979)] and as limited enforceability of contracts with default risk [Eaton, Gersovitz, Stiglitz (1986)]. Costly state verification framework assumes that lenders need to incur a monitoring cost to observe the outcome of an investment. Unless this monitoring cost is incurred, incentive compatibility constraints do not bind and repayment cannot hinge on the outcome. Therefore, incentive compatible contracts are implemented when monitoring takes place in only circumstances where the borrower is unable to comply with the contracted fixed repayment. When the borrowers do not carry out the debt repayment, lenders seize a part the realized outcome by incurring monitoring cost to verify the outcome. Hence, one form of capital imperfections stems from the costly state verification structure. Other form of capital market imperfection depends on the limited enforceability of contracts such that a fraction of total outcome can be confiscated when default takes place. These financial market imperfections raise the cost of financial intermediation. Hence, the borrowers pay higher
than the world interest rate when capital market imperfections in the country are more severe than the rest of the world.

In the model, individuals who do not have enough wealth in the first period to invest on the human capital and to obtain the advanced technology may finance their investment through banking sector. For the sake of simplicity, all financial markets are just represented by the banking sector. The banks are just assumed to play an intermediary role between lenders and borrowers without making excess profit. Therefore, banks borrow with the world interest rate $r$, in international and domestic markets and lend to the investors by incorporating the expected state verification and default cost. The banks, therefore, charge an interest rate, $i$, higher than the world interest rate $r$, as long as expected cost associated with financial market imperfections is positive. The banking sector is assumed to be perfectly competitive and the banks do not default to capture the fact that individuals can lend with interest rate $r$ to the rest of the world without any risk.

Loans are contracted in the first period and repaid in the second period. Since individuals live just two periods, we avoid reputation issue of debt repayment in this overlapping generations setting. Given the option to default, individual $i$, pays the minimum of contracted debt repayment or a fraction of realized output:

$$\min\{\chi Y^S_{t2}; (1 + r^i_d) I_i\}, \quad 0 \leq \chi \leq 1 \quad (4)$$

Where $Y^S_{t2}$ is the income of individual $i$ in period 2 received from investing in the advanced technology. $r^i_d$ is contractual domestic interest rate and determined by the degree of capital market imprecations below and $\chi$ denotes the fraction of individual’s realized output that the banks can appropriate in case of default. $\chi$, therefore, represents
the bank’s bargaining power. $I_i$ is the amount of loan individual $i$ gets from the bank to finance fixed investment $I$, hence, $I_i = I - e_i$. $e_i$ is the amount of wealth or bequest in the first period. Due to the financial market imperfections, borrowing is costly and individuals only borrow enough to invest in the fixed human capital and they lend what is left over from their investment in financial markets with the world interest rate $r$. Similarly, all the wealth that is not invested in the human capital is lent with the interest rate $r$.

In case of default, the banks spend real resources $\eta$ per unit of currency lent to appropriate a fraction of output. This cost is spent for the state verification and enforcement of contract. Hence, the banks expect to receive net debt repayment equal to:

$$E \left[ \min \left\{ \chi Y_{2i}^S - \eta I_i, \ldots, \chi Y_{2i}^S \leq (1 + r_p)I_i \right\} \right]$$

(5)

The threshold levels of idiosyncratic shock associated with default vary in recessions and in booms. More default takes place in the recession than in the booms. In other words, in the booms, aggregate shock is so good that individuals with bad idiosyncratic shocks may find it better not to default. In the recessions, the opposite takes place; aggregate negative shock induces more individual to default even though their idiosyncratic shocks are not too bad.

Let us denote the threshold levels of idiosyncratic shock for default in recessions and booms as $\varepsilon_{R_i}^*$ and $\varepsilon_{B_i}^*$ respectively. Given (1) and (4), the value of $\varepsilon_{R_i}^*$ and $\varepsilon_{B_i}^*$ for individual $i$ can be defined as:

$$(1 - \bar{\delta} + \varepsilon_{R_i}^*)a\chi I = (1 + r_D)I_i$$

(6)
The banking sector is perfectly competitive; hence, there are many banks competing for an individual customer. Moreover, individual customer borrows just enough to carry out the fixed investment. Therefore, the loans demanded by the individuals vary with respect to their initial income. However, the return from investment is same ex-ante for all the individuals. The banks therefore, charge different interest rate to the individuals according to their level of borrowing. It makes sense to think that the expected income in the second period is considered as a collateral that the banks can seize a fraction of it. That means that the amount of collateral is same for all the individuals investing but the amount of loan varies. When the banks compete for individual customer, each bank would reduce the interest rate until the expected cost and benefit are equalized. The expected default for individuals with fewer loans is lower than the expected default of individuals with more loans given that their expected income in the second period is same. The banks then offer lower interest rate to the individuals borrowing less. Therefore, the supply of funds is upward sloping in domestic financial markets due to capital market imperfections associated with default, partial enforcement and state verification costs.

\[(1 + \delta + \varepsilon^*_a) a \chi I = (1 + r_D) I, \quad (7)\]

or

\[
\varepsilon^*_a = \max \left[ -\tilde{\varepsilon}, \min \{\bar{\delta} - 1 + \frac{1 + r_D^I I}{a \chi I}; \tilde{\varepsilon} \} \right] \quad (8)
\]

\[
\varepsilon^*_b = \max \left[ -\tilde{\varepsilon}, \min \{-(\bar{\delta} - 1) + \frac{1 + r_D^I I}{a \chi I}; \tilde{\varepsilon} \} \right] \quad (9)
\]
The expected return on typical bank’s lending to individual $i$ per unit of loan in booms and recessions are:

$$E(R_B) = 1 + r = \int_{\epsilon_{R_h}}^{\bar{\epsilon}} (1 + r_D') f(\epsilon_i) d\epsilon_i + \int_{-\bar{\epsilon}}^{\epsilon_R} \frac{a_1 I}{I} (1 + \bar{\delta} + \epsilon_i) f(\epsilon_i) d\epsilon_i - \eta \int_{-\bar{\epsilon}}^{\epsilon_R} f(\epsilon_i) d\epsilon_i \quad (10)$$

$$E(R_R) = 1 + r = \int_{\epsilon_{R_h}}^{\bar{\epsilon}} (1 + r_D') f(\epsilon_i) d\epsilon_i + \int_{-\bar{\epsilon}}^{\epsilon_R} \frac{a_1 I}{I} (1 - \bar{\delta} + \epsilon_i) f(\epsilon_i) d\epsilon_i - \eta \int_{-\bar{\epsilon}}^{\epsilon_R} f(\epsilon_i) d\epsilon_i \quad (11)$$

Since the probability of the next period to be boom or recession episode is $1/2$, we can write the bank’s expected yield for unit of loan as:

$$E(R) = 1 + r = \begin{pmatrix}
+1/2 \int_{\epsilon_{R_h}}^{\bar{\epsilon}} (1 + r_D') f(\epsilon_i) d\epsilon_i + \int_{-\bar{\epsilon}}^{\epsilon_R} \frac{a_1 I}{I} (1 + \bar{\delta} + \epsilon_i) f(\epsilon_i) d\epsilon_i \\
+1/2 \int_{-\bar{\epsilon}}^{\epsilon_R} \frac{a_1 I}{I} (1 - \bar{\delta} + \epsilon_i) f(\epsilon_i) d\epsilon_i + \int_{\epsilon_{R_h}}^{\bar{\epsilon}} f(\epsilon_i) d\epsilon_i \\
-1/2 \eta \int_{-\bar{\epsilon}}^{\epsilon_R} f(\epsilon_i) d\epsilon_i + \int_{-\bar{\epsilon}}^{\epsilon_R} f(\epsilon_i) d\epsilon_i
\end{pmatrix} \quad (12)$$
The first term in (12) indicates that the banks receive \((1+r^i_D)\) in case of no default. But there is a possibility that the individual i’s idiosyncratic shock falls into the default region. The second term indicate the amount of appropriation associated with partial debt repayment in case of default. The last term is the monitoring cost incurred to verify the output after default.

The aggregate volatility in this model is characterized as the magnitude of \(\delta\). For higher values of \(\delta\), the output of the economy changes more between recessions and booms. It can be see from (6) and (7) that for a low enough degree of volatility, no default takes place such that even in recessions with the worst idiosyncratic shock, the individuals have an incentive to pay fully if:

\[
(1-\bar{\delta} - \bar{\epsilon})a\chi I \geq (1+r^i_D)I_i \quad (13-1)
\]

or

\[
\bar{\delta} \leq (1-\bar{\epsilon}) - \frac{(1+r^i_D)I_i}{a\chi I} \quad (13-2)
\]

For individuals financing all the fixed investment from banking sector, \(I_i = I\), the condition for no default even in recessions and with the worst idiosyncratic shock is:

\[
\bar{\delta} \leq \delta^* = (1-\bar{\epsilon}) - \frac{(1+r^i_D)}{a\chi} \quad (13-3)
\]

If individuals with the highest debt and with the worst idiosyncratic shock do not default in recessions, nobody defaults in the economy. Equation (13-3) states that for level of volatility below \(\delta^*\), nobody defaults and hence there is no capital market imperfections associated with default. Consequently, the domestic interest rate is equal to the world interest rate:
\[ r_d^i = r \text{ if } \tilde{\delta} \leq \delta^* = (1 - \tilde{\varepsilon}) - \frac{(1 + r_d^i)}{a\chi} \]  (13-4)

Equation (13-4) also implies that an increase in \( r \) makes the economy more vulnerable to the aggregate shocks such that smaller shocks are enough to generate capital market imperfections.

When (13-1) is reversed, the bank’s return in recessions and booms would differ because of the default and partial repayment. In this model, we are interested in volatility when the financial market imperfections exist. Following proposition shows that the interest rate charged by the banks increases when volatility exceeds a certain threshold where nobody defaults in the booms.

**Proposition 12:**

Greater volatility of aggregate macroeconomic shocks increases the financial imperfections associated with costly state verification and default and in turn increases the domestic interest rate charged to individual \( i \) by the banks \((r_d^i)\) when

\[ \tilde{\delta} \geq \delta^{**} = -(1 - \tilde{\varepsilon}) + \frac{(1 + r_d^i)}{a\chi}. \]

When volatility changes in the range \((\delta^{*} \leq \tilde{\delta} \leq \delta^{**})\), greater volatility does not lead to higher financial imprecations and higher \((r_d^i)\) because the expected loss from defaults remain constant.

The volatility and capital market imperfection follow a nonlinear path. For low level of volatility there is no capital market imperfection and the domestic interest rate is equal to the world interest rate. For an intermediate degree of volatility, capital market

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\(^2\) Proofs are delegated to the appendix.
imperfections and the resulting domestic interest rate are constant. After a certain threshold ($\delta''$), greater volatility increases the financial imperfections and the domestic interest rate.

![Graph showing relationship between $r_D$ and $r$](Figure 2)

**Proposition 2:**

(i) Higher intermediation cost associated with state verification cost increases the domestic interest rate $\frac{\partial(1+r_D)}{\partial \eta} \geq 0$.

(ii) Greater macroeconomic volatility aggravates the capital market imperfections associated with costly intermediation $\eta_i$, $\frac{\partial^2(1+r_D)}{\partial \eta \partial \delta} \geq 0$ when $\frac{\partial^2(r_i^*)}{\partial \delta^2} \geq \left| \frac{\partial^2(r_i^*)}{\partial \delta^2} \right|$ or equivalently $\delta \geq \delta''$.

### 3-Volatility and Growth

Decision to invest in advanced technology depends on the initial wealth of an individual when capital markets are imperfect. When there is no capital market imperfection, individuals lend with world interest rate ($r$) and as long as the expected net
outcome from investing in better technology exceeds the expected net outcome from inferior technology, everybody invests in human capital. However, in the presence of capital market imperfections associated with state verification cost and partial enforcement of debt repayment, the equilibrium interest rate in the country exceeds the world interest rate. Moreover, individuals differ with respect to their initial endowment (bequest from their parents) and therefore need different levels of loan to finance their investment on human capital. First, let us consider an individual with initial wealth \( e_i \) higher than fixed cost of investment, \( I \). This individual does not need to borrow from the financial markets and incur additional cost for financing the fixed investment, \( (r^D_i - r) \). This individual invests in the advanced technology and lends what is left over from his/her investment with interest rate \( r \). Then, this individual’s expected lifetime income at the end of the second period is:

\[
W_i^S = E(Y_{2i}^S) + (e_i - I)(1 + r) \quad (e_i \geq I) \quad (1)
\]

Now let us consider an individual with initial wealth less than \( I \). This individual either does not invest in the advanced technology or borrow in financial markets to invest in fixed investment \( I \). When an individual who inherits an amount of \( e_i \) in the first period of life chooses not to invest in the advanced technology, his/her expected lifetime income is:

\[
W_i^N = \left( EY_{1i}^N + e_i \right)(1 + r) + EY_{2i}^N \quad (2)
\]

An alternative for this individual is to borrow \( (I - e_i) = I_i \) and to invest in the advanced technology. His/her lifetime income is:

\[
W_i^{SF} = E(Y_{2i}^S) - \min \left( \begin{array}{c} 0 \end{array} ', (1 + r^D_i)(I - e_i) \right) \quad (3)
\]
by using (1), (3) can be rewritten as:

\[ W_i^{SF} = aI - 1/2(1 + r_D^f)I_i \left[ \int_{\xi_i}^{\bar{\xi}} f(\xi_i) d\xi_i + \int_{\xi_i}^{\bar{\xi}} f(\xi_i) d\xi_i \right] - 1/2aR^I_i \left[ \int_{-\bar{\xi}}^{-\xi_i} f(\xi_i) d\xi_i + \int_{-\bar{\xi}}^{-\xi_i} f(\xi_i) d\xi_i \right] \]

where \((aI)\) is expected outcome from investing and the second part is the cost of financing and the third part is the cost of financing in the case of default.

It is clear that when \( W_i^{SF} \geq W_i^N \) all the individuals prefer to invest in the advanced technology. \( W_i^{SF} \geq W_i^N \) can be rewritten as:

\[ e_i^* \geq \frac{E Y_i^N (2 + r) - \left\{ aI - \left( 1/2I(1 + r_D^f)C - 1/2aR^D \right) \right\}}{1/2I(1 + r_D^f)C - (1 + r)} \]

where

\[
\begin{align*}
\left[ \int_{\xi_i}^{\bar{\xi}} f(\xi_i) d\xi_i + \int_{\xi_i}^{\bar{\xi}} f(\xi_i) d\xi_i \right] &= C \\
\left[ \int_{-\bar{\xi}}^{-\xi_i} f(\xi_i) d\xi_i + \int_{-\bar{\xi}}^{-\xi_i} f(\xi_i) d\xi_i \right] &= D
\end{align*}
\]

Individuals with initial wealth less than \((e_i^*)\) prefer not to invest in the advanced technology. Therefore, capital market imperfections prevent the individuals with income less than \((e_i^*)\) to benefit from better technology due to higher interest rate to the borrowers. Therefore, initial distribution of income partially determines whether the individuals would benefit from better technology. Countries with more equal income distribution suffer less from capital market imperfections and invest more in better production technologies and grow faster.
Next proposition highlights the effects of volatility on the threshold level of initial endowment ($e^*$).

**Proposition 3:**

*Greater volatility increases the gap between domestic and international interest rate and prevents more people from benefiting from better technology and consequently reduces the growth for $\delta \geq \delta^*$. 

\[
\frac{\partial e^*_i}{\partial \delta} \geq 0 
\]

The model suggests that economic volatility is distributed asymmetrically among individuals in the society. The rich who have enough wealth to invest in the advanced technology (or human capital) are not affected by increasing volatility. However, the poor are unable to invest on human capital due to increasing financial imperfections associated with greater volatility.

**4- Conclusion**

This paper provides a model to account for the empirical evidence that volatility reduces growth. In the model, greater volatility increases the cost associated with capital market imperfections and induces the financial intermediaries to charge higher interest rates. The model is based on one of overlapping generations with two types of technologies. The more productive technology requires fixed investment in the first period. Individual with income less than the amount of fixed investment may borrow in financial markets to obtain more productive technology. Increase in volatility raises the cost of borrowing and makes it less attractive to invest in more productive technology for
individuals below certain income in the first period. Hence, volatility reduces growth by deterring people from taking advantage of more productive technology. This model also explains the empirical findings of Ramey and Ramey (1995) that investment is not the channel between volatility and growth by suggesting that totals factor productivity rather than the total factor accumulation is the key for growth.
References


Appendix

Proof of Proposition 1:

This proposition follows from the fact that the banks are risk neutral and make no excess profit. Therefore, the expected return in (12) must be equal to a constant \((1 + r)\). In the boom, the condition that even the most indebted individual, \((I_i = I)\) with the worst idiosyncratic shocks does not default:

\[ (1 + \tilde{\delta} - \tilde{\varepsilon})a\chi I \geq (1 + r_D)I_i \]

equivalently,

\[ \tilde{\delta} \geq \delta^{**} = -(1 - \tilde{\varepsilon}) + \frac{(1 + r_D)}{a\chi} \]

For \( \tilde{\delta} \geq \delta^{**} \), since no default takes place in booms, an increase in aggregate volatility only increase the number of individual defaults in recession and the total cost associated with defaults. Higher cost then requires the banks to increase \((r_D)\) such that zero profit condition is satisfied. Let us take the derivative of (12) with respect to \((\tilde{\delta})\)

\[ (P-1) \]

\[ \frac{\partial E(R)}{\partial \tilde{\delta}} = \frac{\partial (1 + r)}{\partial \tilde{\delta}} = 0 = \int_0^{\tilde{\varepsilon}} a\chi f(\varepsilon_i)d\varepsilon_i - \int_0^{\tilde{\varepsilon}} \frac{a\chi f(\varepsilon_i)}{I_i}d\varepsilon_i - \eta \left[ \frac{\partial \hat{E}_B^*}{\partial \tilde{\delta}}F(\varepsilon_R^*) + \frac{\partial \hat{E}_R^*}{\partial \tilde{\delta}}F(\varepsilon_R^*) \right] \]

\[ \frac{\partial \hat{E}_B^*}{\partial \tilde{\delta}} = \frac{\partial \hat{E}_R^*}{\partial \tilde{\delta}} = 0 \text{ and } F(-\tilde{\varepsilon}) = 0 \text{ we can rewrite (P-1) as :} \]
since $\overline{\varepsilon} \geq \varepsilon_{R_1}^* \geq \varepsilon_{R_2}^*$, the second term in (P-2) is $\leq 0$. The first term in (P-2),

$$\left(\frac{\partial \varepsilon_{R_1}^*}{\partial \overline{\delta}}\right) \left[ - (1 + r'_D) + \left\{ \frac{a\chi l}{l} \right\} \right] \leq 0$$

putting

$$\varepsilon_{R_1}^* = \max \left\{ -\overline{\varepsilon}, \min\{\overline{\delta} - 1 + \frac{1 + r'_D}{a\chi l} ; \overline{\varepsilon} \} \right\}$$

into the first term gives:

$$\left\{ - (1 + r'_D) \left[ 1 - \int_{\overline{\varepsilon}} f(\varepsilon_i) d\varepsilon_i \right] - \eta F(\varepsilon_{R_1}^*) \right\} \leq 0 ; \text{ because } \left[ 1 - \int_{\overline{\varepsilon}} f(\varepsilon_i) d\varepsilon_i \right] \geq 0 .$$

Since the first and the second terms are $\leq 0$ then the last term in (P-2) must be $\geq 0$ and

$$\frac{\partial (1 + r'_D)}{\partial \overline{\delta}} \geq 0 .$$

Therefore, we can conclude that for a given level of volatility when greater volatility ($\overline{\delta} \geq \delta'''$) requires higher domestic interest rate $QED.$
Proof of Proposition 2:

(i) by differentiating (12) with respect to $\eta$, one gets:

$$\frac{\partial E(R)}{\partial \eta} = \frac{\partial (1 + r)}{\partial \eta} = 0 = \frac{\partial (1 + r_D^i)}{\partial \eta} - 1/2(\int_{-\varepsilon}^{\varepsilon} f(\varepsilon_i)d\varepsilon_i + \int_{-\varepsilon}^{\varepsilon} f(\varepsilon_i)d\varepsilon_i)$$

Since the second term is $\geq 0$, $\frac{\partial (1 + r_D^i)}{\partial \eta} \geq 0 \textbf{ QED.}$

(ii) $\frac{\partial^2 E(R)}{\partial \eta \partial \delta} = 0 = \frac{\partial^2 (1 + r_D^i)}{\partial \eta \partial \delta} - 1/2(\frac{\partial \varepsilon^*_R}{\partial \delta}(F(\varepsilon^*_R))$

since the second term $1/2(\frac{\partial \varepsilon^*_R}{\partial \delta}(F(\varepsilon^*_R))$ is $\geq 0$, $\frac{\partial^2 (1 + r_D^i)}{\partial \eta \partial \delta} \geq 0 \textbf{ QED.}$